CONTROLLER DESIGN FOR A TWO-LINK ROBOTIC MANIPULATOR SUBJECTED TO EXTERNAL DISTURBANCES AND PARAMETRIC VARIATIONS

A Thesis

Submitted in partial fulfillment of the

requirement for the award of the degree of

MASTER OF TECHNOLOGY

IN

ELECTRICAL ENGINEERING

(CONTROL SYSTEMS)

BY

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ABSTRACT

A robust sliding mode control (SMC) algorithm is designed which is used to control a 2-link robotic arm. The controller is tested for various types of disturbances and model parametric uncertainties. The novelty of the work lies in the fact that the designed controller is capable of handling slow varying disturbances, fast varying as well as unpredictable disturbances. Simulation results validate the accurate tracking capability and robust performance.

ACKNOWLEDGEMENT

It is with immense respect and gratitude that I acknowledge the support and help of my advisor, Dr. (Mrs.) Sarbani Chakraborty, Department of Electrical and Electronics Engineering for her wholehearted and invaluable guidance for the successful completion of this project. Without her sustained and sincere efforts, this project would not have taken shape. She encouraged and helped me to overcome the various difficulties that I faced at various stages of this course and her continuous guidance and assessment of this work has imparted in me, a deep understanding of this subject.

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I am grateful to the faculty and staff of Birla Institute of Technology, Mesra for their direct and indirect support and for the immense learning opportunities.

I will remain deeply indebted to my parents, my family and friends for their continued love and support. I will also remain in debt to my alma matter for making what I am.

Date - PARAJ GANCHAUDHURI
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LIST OF ABBREBIATIONS

VSCS	Variable Structure Control System
SMC	Sliding Mode Control
LTI	Linear time Invariant
TDC	Time Delay Control
SMCPE	Sliding Mode Control with Perturbation Estimation
SO	Sliding Observer
LQR	Linear Quadratic Regulator
PD	Proportional Derivative
PID	Proportional Integral Derivative
NDO	Non-linear Distance Observer
ANOVA	Analysis of Variance
ABC	Artificial Bee Colony
SCARA	Selective Compliance Assembly Robot Arm
MIMO	Multiple Input Multiple Output
DOF	Degree of Freedom
UDE	Uncertainty and Disturbance Estimator
SMCPE	Sliding Mode Control with Perturbation Estimation
CG	Center of Gravity

Department of Electrical and Electronics Engineering

Birla Institute of Technology, Mesra, Ranchi - 835215 (India)

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MASTER OF TECHNOLOGY (M. TECH) IN CONTROL SYSTEMS

Course Name: M. TECH THESIS **Credits:16.0**

Course Code: EE650

Duration: Semester IV

Project title:

Controller Design for a Two-Link Robotic Manipulator Subjected to External Disturbances and Parametric Variations

Pre-requisite(s):

Non-linear control system, Lagrangian Formulation, Linearization, State Space Modeling, Sliding Model Control, Sliding Surface, MATLAB, SIMULINK.

Topic Covered:

Nonlinear control system, Control system design, Dynamic analysis of Robotic manipulator, Sliding Mode Control, Chattering in Sliding Mode Control, Parametric Uncertainty in nonlinear systems, Computed Torque Systems

Course Evaluation Methodology:

Periodic assessment by review during semester based on presentation and viva; end-semester assessment by external expert (presentation and viva), thesis evaluation.

Course Objectives:

The main objectives of the research work carried out in this thesis are as follows:

A. Study of dynamical model of a two degree of freedom robotic manipulator.

B. Integrate a sliding mode controller to this system based for trajectory tracking problem by choosing a sliding manifold based on the error dynamics and ensure that chattering is overcome.

C. Test the system against various types of trajectory tracking problems keeping in account of the actuator efforts and smoothness of actuator efforts and against various types of disturbances and uncertainties acting on the system.

Course Outcomes (COs):

On completion of this course, the student will be able to:

CO1: Obtain the mathematical model of the two-link robotic manipulator and understand the various dynamics acting on it.

CO2: Build sliding mode control algorithm and integrate it with the robotic manipulator for trajectory tracking problems.

CO3: Analyze how the robotic manipulator responds to various types of commands and estimate the controller gains and required actuator efforts.

CO4: Understand how the robotic manipulator responds to different types of disturbances and parametric uncertainties acting on it.

CO5: Get an idea how the physical plant will require actuator efforts for different robotic tasks and how do choose the right controller gains for different actions.

Program Educational Objectives (PEOs):

The program educational objectives for M. Tech. (Control Systems) are as follows:

PEO1: To acquire in-depth knowledge of complex Electrical Engineering problems especially in Control Systems to impart ability to discriminate, evaluate, analyze critically and synthesize knowledge pertaining to state of art and innovative research.

PEO2: To solve complex control system problems with commensurate research methodologies as well as modern tools to evaluate a broad spectrum of feasible optimal solutions keeping in view socio- cultural and environmental factors.

PEO3: To possess wisdom regarding group dynamics to efficaciously utilize opportunities for positive contribution to collaborative multidisciplinary engineering research and rational analysis to manage projects economically.

PEO4: To communicate with engineering community and society at large adhering to relevant safety regulations as well as quality standards.

PEO5: To inculcate the ability for life-long learning to acquire professional and intellectual integrity, ethics of scholarship and to reflect on individual action for corrective measures to prepare for leading edge position in industry, academia and research institutes.

Program Outcomes (POs):

The program outcomes for M.E. (Control Systems) are as follows:

PO1: An ability to independently carry out research /investigation and development work to solve practical problems.

PO2: An ability to write and present a substantial technical report/document.

PO3: Students should be able to demonstrate a degree of mastery over the area as per the specialization of the program. The mastery should be at a level higher than the requirements in the appropriate bachelor program.

PO4: Recognize the need for continuous learning and will prepare oneself to create, select and apply appropriate techniques and modern engineering and IT tools to solve complex control system problems.

PO5: Demonstrate knowledge of engineering and management principles and apply to manage projects efficiently and economically with intellectual integrity and ethics for sustainable development of society.

PO6: Possess knowledge and understanding to recognize opportunities and contribute to collaborative-multidisciplinary research, demonstrate a capacity for teamwork, decision-making based on open-mindedness and rational analysis in order to achieve common goals.

Mapping between Course Objectives and Course Outcomes (COs):

Course Code/ Name	Course Outcomes / Course Objectives	CO1	CO2	CO3	CO4	CO5
EE650/ Thesis	1	H	H	H	H	M
	2	L	H	M	H	H
	3	M	M	H	M	M
	4	L	H	L	H	H

H=high, M= medium, L= low

Mapping between Course Outcomes (COs) and Program Outcomes (POs):

Course Code/ Name	POs / COs	PO1	PO2	PO3	PO4	PO5	PO6
EE650/ Thesis	CO1	H	H	M	H	NA	M
	CO2	H	M	L	L	L	M
	CO3	H	H	M	H	NA	M
	CO4	M	H	H	H	L	H
	CO5	M	H	M	H	NA	H

H= high, M= medium, L= low, NA= not applicable

Mapping between Program Educational Objectives (PEOs) and Program Outcomes (POs):

Course Code/ Name	POs / PEOs	PO1	PO2	PO3	PO4	PO5	PO6
EE650/ Thesis	PEO1	H	H	H	H	M	M
	PEO2	H	H	H	H	H	M
	PEO3	H	H	H	H	H	H
	PEO4	H	H	H	H	H	H
	PEO5	H	H	H	H	H	H

H= high, M= medium, L=low

CHAPTER 1
INTRODUCTION

1.1 BACKGROUND

A robotic Manipulator is a type of artificially created mechanical arm, which is programmed and tuned to function like a human arm and perform complex and iterative tasks quickly and efficiently. The arm may be a solo mechanism or may be part of a more complex robot. The links of such a manipulator are connected by actuating joints that allow either rotational motion (such as in an articulated robot) or translational (linear) displacement.

Control of robot manipulators is still a challenging control systems design problem today due to its high nonlinearity and strongly coupled robot dynamics [1]. The task gets even more complicated when robotic manipulators are subjected to various unknown environments in the form of model uncertainties and unmeasurable external disturbances.

Robotic Manipulators find applications in critical fields like surgery, nuclear containments, industrial assembly lines etc., welding, repairing pipelines on the ocean floor, remote servicing of utility power lines, industrial assembly lines in putting cars together, medical assistance to patients during infectious diseases and so accurate control of the robot arms has become an important requirement. So, the figure of merit defining the controller is robustness and the modern-day design demands optimal control with minimal effort or energy to achieve a particular work. The end effects of the manipulators are to track some desired trajectories as close as it is possible. So, trajectory-tracking problem is the most important and effective test to grade a controller design. A detailed study of various strategies suggested in the literature for the design and implementation of robust controllers of robotic manipulators is mentioned in [2].

Ideally, the most common control systems design formulation is the systematic assignment of the poles of the system of a set of de coupled and linearized sub-system based on certain specifications. The essential requirements for such designs are that the system nonlinearities are neglected and the controller is acted upon with the most accurate dynamical model. Any deviation due to mismatching parameter or modelling structured and unstructured uncertainties will degrade the performance of the system and undermine the benefits of a controller. In most cases the uncertainty is considered to be limited by equations having higher order polynomials or approximated by some continuous functions.

Various model based robust control techniques have been proposed in the available literature for synthesizing and designing controllers for robotic manipulators. Some notable controllers include algorithms based on Proportional-Integral-Derivative, Lyapunov-based theory, optimal control, fuzzy logic controllers, neural networks etc.

Sliding Mode Control or SMC is a non-linear and robust control methodology. The SMC based controller is resistant to unknown uncertainties and parameter variations (if any), and do not ask for an accurate model of the manipulator, therefore an ideal choice for controlling the system of interest in this article. The SMC technique utilizes the theory of Variable Structure System (VSS) [33]. It involves driving an underlying state or the error dynamics of a state to be maintained at an attractive manifold, also called a sliding manifold due to which, the desired dynamic behavior of the system can be asymptotically ascertained. The proceeding for a sliding mode control action involves two phases, the reaching phase and the sliding phase [31]. The reaching phase tries to align the required system state approach towards the sliding manifold while the sliding phase ensures the required dynamic behavior is achieved. During this sliding motion, the dynamics or properties of the system becomes invariant, thus making it less important to neutralize the system nonlinearities, which are otherwise required to be nullified if a conventional controller is used. The yielding dynamic motion of the states also becomes immune to certain parameter variations and external disturbances provided there are known bounds in those disturbances and variations [35]. However, the robustness characteristics of the conventional sliding mode control with respect to alterations of system parameters and external perturbations can only be ensured after the condition of sliding phase is achieved. During the reaching phase, there isn't any guarantee for robustness [44]. An ideal control action from a sliding mode controller results in a discontinuous control action thus reflecting in a discontinuous actuator effort called chattering, which may demand high frequency switching actuators. The chattering phenomenon can be substantially mitigated by using a linear saturation boundary layer function in the SMC algorithm [42]. One more advantage of SMC is that we can obtain sliding mode motion for complex order systems even if the controller is obtained with reduced order and simpler modelling [45].

1.2 MOTIVATION

The motivation behind this research is because of the challenges in controlling the robotic manipulator which are as follows –

- Robotic manipulators are complex systems having high dynamic coupling. They are time-varying and inherently nonlinear systems.
- The control effort which is torque at the joints are coupled.
- The manipulators are usually subjected to both structured as well as unstructured parametric uncertainties which makes the precise and accurate position control a complicated task.
- The end effects of the robotic manipulators are accurate tracking against a command which is the most important and significant task of a robotic manipulator.
- The traditional control algorithms are inadequate under large scale uncertainty and have unprecedented variations in system parameters.
- Obtaining a perfect and accurate mathematical model is a challenge and becomes difficult in case of greater number of links and joints in a complicated robotic arm.

1.3 THESIS OBJECTIVE

The main objectives of this research work are as follows

1. Study of dynamical model of a two degree of freedom robotic manipulator.
2. Integrate a sliding mode controller to this system based for trajectory tracking problem by choosing a sliding manifold based on the error dynamics and ensure that chattering is overcome.
3. Test the system against various types of trajectory tracking problems keeping in account of the actuator efforts and smoothness of actuator efforts and against various types of disturbances and uncertainties acting

on the system.

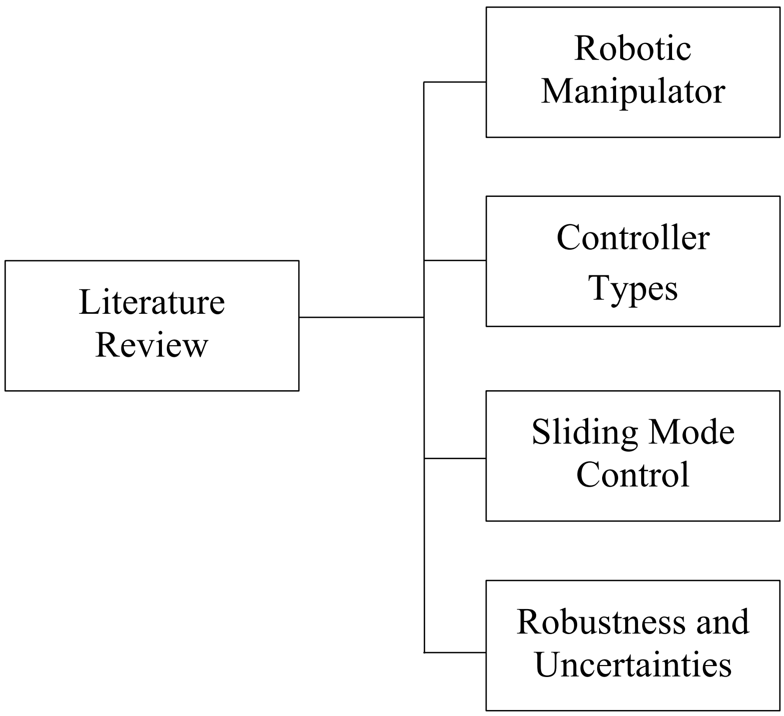
1.4 THESIS ORGANIZATION

The organization of this research article is as follows. In Chapter 2, a literature survey is done on robotic manipulators, nonlinear control techniques, sliding Mode Controllers and disturbance and parametric uncertainties on manipulators. The mathematical formulations for the dynamics of a 2-link robotic arm is reviewed in Chapter 3. Chapter 4 developes a Sliding mode control algorithm and its integration with the system. Chapter 5 highlights the methodologies for parametric uncertainties on robotic manipulators. In Chapter 6, the proceedings for simulations are provided followed by the results and discussions. Finally, Chapter 7 concludes the findings of this work and also mentions some future scope.

CHAPTER 2

LITERATURE SURVEY

This chapter presents a literature survey conducted in certain aspects that cover all the essential aspects of this project. The survey includes topics like the Robotic manipulators, their advent and development over time. The survey also shows comparison and analysis of various controller techniques, and how one outperforms the another and in which respect. Further, there is extensive survey on the Sliding Mode Controller. Lastly, we cover Uncertainty and Robustness aspects of the robotic manipulator and the controller.



2.1 LITERATURE REVIEW ON ROBOTIC MANIPULATORS

M. Vidyasagar in [1] provided the basic framework of dealing with physical systems like robotic manipulators. It highlights how physical systems with joints can be transcended into mathematical equations and how classical mechanics can be calibrated using controller designs.

Abdallah C, Dawson D, Dorato P and Jamshidi M in [2] summarises the available literature on approaches to robust control of rigid robots. They are categorized into five major design approaches : linear-multivariable approach, the passivity approach, the variable-structure approach, the saturation approach, and the robust-adaptive approach.

Elisha D. Markus, Adisa A. Jimoh and John T. Agee in [3] describes the application technique of differential flatness for trajectory tracking of a robot manipulator in the presence of gravity and friction which otherwise increases the complexity in the coupled nonlinear dynamics of the manipulator. This method helps in obtaining ideal trajectories and resulting accurate trajectory tracking.

Kaung Khant Ko Ko Han, Theingi and Aung Myo Thant Sin in [4] performs the dynamic and kinematic analysis of two link robot arm in a vertical movement. The derivation of the Newton-Euler method and Lagrange- Euler are also verified. Joint torques calculated in each time interval are plotted and actuated using a proposed (PID plus friction compensator) controller. The friction compensator is integrated with Coulomb friction, Static friction, Stribeck effect and Viscous friction.

B. Bona and M. Indri in [5] studies the performances of some linear and nonlinear observers by experimenting on the controllability of a SCARA two-link manipulator with multiple revolute joints. The analysis is carried out by using the observed velocities to implement a linear state feedback structure for the robot control. The results derived shows that different aspects (for example the required action, the knowledge of the robot dynamic model) may influence the selection of a given observer.

S. Nicosia and P. Tomei in [6] have derived a dynamic output feedback controller for elastic joint robots where the only measurable quantities are link positions and the obtained controller is also able to track the trajectories having any arbitrary initial position. The controller consists of a non-linear observer which guarantees asymptotic stability for the error-dynamics. The Controller guarantees high degree of robustness with respect to parametric variations and its convergence is locally exponential.

A. Green and J. Z. Sasiadek in [7] have found the solutions to operational problems with robot manipulators on structural flexibility and subsequent difficulties with their position control initially with the help of an LQR algorithm and fuzzy logic algorithms dictated by a Jacobian transpose control law. Secondly, a time delay provided does not result in a minimum phase system. Finally, a fuzzy logic system adapts the control law in response to elastic deformation inputs. Results obtained show higher trajectory tracking accuracy as compared to a rigid dynamic robot.

F. Reyes and R. Kelly in [8] compares four model-based control structures with a direct-drive robotic manipulator. They obtain outcomes for the following controllers: Computed-Torque controller, Proportional Derivative + control, Proportional Derivative control with calculated feed forward and Proportional Derivative control. All controllers have considered friction compensation as well as cancellation of gravitational torques and have been evaluated with a performance index for trajectory tracking problems.

M W Spong in [9] uses the Lyapunov-based theory of guaranteed stability of uncertain systems on a n-link robotic manipulator and derives a robust non-linear control law and proves that uniform ultimate boundedness of the tracking error depends only on the inertial parameters. The control law obtained does not require any previous assumptions regarding closeness in norm calculation of the calculated inertia matrix to the original inertial matrix.

Chen WH, Gawthrop PJ, Ballance DJ and O'Reilly J. in [10] have developed a new nonlinear disturbance observer (NDO) whose exponential stability is guaranteed on the maximum velocity and the parameters of the robotic manipulators. The observer does not require any linearized model and its performance is verified using a 2-link robotic manipulator with friction estimation and compensation.

M. Plooij, W. Wolfslag, and M. Wisse in [11] have considered a feed-forward controller where the final position of motion is robust to any uncertainty in the friction model of the robotic manipulator using rest-to-rest motions of robotic arms. They have considered Viscous, Coulomb and torque dependent friction on a single link robotic arm, of which analytical, simulation and hardware results were obtained.

Sage HG, Ostertag E. and Mathelin MFD in [12] presents an overview of the textbook robust control structures for manipulator systems. This overview also includes mathematical references of actuator dynamics and joint flexibility. The different control structures are categorized as follows: linearized structures, passivity-oriented schemes, Lyapunov-based structures, SMC schemes, non-linear H omega schemes and adaptive and control structures.

Zhao D, Li S, Zhu Q, Gao F. in [13] have used finite-time Lyapunov stability principle and developed a new robust finite-time stability control approach for robotic manipulators and proved their design with backstepping method. For theoretical understanding, a corresponding stability analysis is presented to highlight the underlying design issues as well as safe operation of a two-link robot model.

B.K. Rout and R.K. Mittal in [14] have optimised the parametric designs for a 2-link robotic manipulator. Different values of parameters are considered to get a wide overview of the optimization algorithm. ANOVA technique is used and the data for various parameters is used to analyse the statistical significance of kinematic and dynamic parameters on performance of manipulator using.

2.2 LITERATURE REVIEW ON CONTROLLER TECHNIQUES

Nasr A. Elkhateeb, Ragia I. Badr in [15] optimized a PID controller on the dynamic inertia weighing Artificial Bee Colony algorithm

(ABC) for a robotic manipulator. There is a coupled relationship between the actuator efforts and the position as well as acceleration of the robot arm. The designed algorithm optimizes the end effector for a time dependent input and turns the controller robust enough to deal with external disturbance.

J. P. P., J. P. Perez, A. Flores, J. L. Meza in [16] uses adaptive neural network for controlling a robotic manipulator. It is based on recurrent neural networks and Lyapunov functions methodology and Proportional-Integral-Derivative (PID) control for nonlinear systems. The control law is based on PID approach and controller structure is composed of a neural identifier. To verify the analytical results, an experiment of dynamical network is simulated and a theorem is proposed to ensure the tracking of the nonlinear system.

Alvarez-Ramirez, Ilse & Kelly, Rafael and Jose & Cervantes in [17] obtains a Nobel PID controller based on modelling error compensation. It is proved that semiglobal PID stabilization and the Proportional Integrate Derivative control law stability is dependent on only the inertial parameters of the robotic manipulator. It is also proved that PID control with high gain limit can recover the performance of inverse dynamics control. The author presents a easy tuning guideline derived from the above mentioned PID control structure and stability of the closed-loop system.

Stout WL, Sawan ME in [18] develops a H∞ controller for flexible joints of an industrial robotic manipulator. They have obtained two design methodologies i) mixed-H∞ design, and ii) H∞ design with loop shaping. Out of the two controller structures, one uses only the actuator location, while the other utilizes the actuator location and end effector acceleration. So, the resulting controllers are compared to a standard controllers and choice of weighting functions are derived. They have also discussed model order reduction for controllers.

Lin F and Brandt RD in [19] presents a new optimal control approach where uncertainties in terms of unknown loadings are overcome. They have designated performance indices and optimised with reference to those indices to achieve robust control. They have verified their proceedings using a two-joint SCARA type robot, by obtaining Robust control solution using Riccati equations.

Ghorbel F, Hung JY and Spong MW in [20] have shown that if a correlation term is added to the control law and oscillations are damped out at the joints having weak joint elasticity, a singular perturbation argument can make the flexible joint robotic manipulators have adaptive capability and become insensitive to parametric uncertainty.

A-V. Duka in [21] complex structural behaviour of fabric materials and geometric nuances in the robotic structures results in imprecise sewing of fabrics. To prevent such deviation from sewing trajectories, a neural network-based design for obtaining the positional error rectification of a 2-R manipulator is proposed. Geometrical errors of link longitudes on positional accuracy have been simulated and investigated. Performance of the proposed approach have thus been experimentally verified.

J. Wilson, R. Dubay and M. Charest in [22] have carried their research on three different nonlinear and model predictive algorithms which are adaptive and non-linear model predictive control (nMPC), Proportional Integrate Derivative based nMPC (PIDnMPC), and a novel simplified nMPC (SnMPC) and have simulated and verified them experimentally using a fabricated planar two link vertical robotic arm apparatus. Trajectory tracking, computational complexity and transient response dynamics are the aspects on which they have made these controllers compete with eachother.

O. Djaneye-Boundjou and R. Ordóñez in [23] have revisited the design cycle for the Proportional-Integral-Derivative (PID) for torque control of robotic manipulators, when there is an increase in (DOF) and/or in the case of designing a Multi-Input Multi-Output (MIMO) PID controller. They have utilised a stable Adaptive Particle Swarm Optimizer to tune the P-I-D gain matrices, based on a cost function considering various aggregated performance indices.

F. L. Lewis in [24] have used a neural network controller various attributes of a robotic manipulator like for position control, force control, parallel-link mechanisms, and digital neural network control and has proved that his model independent controller is superior to adaptive controllers.

R. Kelly in [25] proposed a controller using integral of a nonlinear function and feedbacking a linear proportional–derivative function. The author has described few regulator gains based on a class of some non-linear functions by using Lyapunov's direct method and LaSalle's to ensure global and asymptodic stability.

A. Zavala-Rio and V. Santibanez in [26] have carried their research on robotic manipulators having input saturations. They have provided alternate approaches which are simple extensions of the PD-with-gravity-compensation (PDgc). Moreover, their algorithm has a better approach to a PDgc control signal compared to other algorithms of the same genre.

C. Sun, W. He, H. Gao and Y. Yu in [27] based their dynamical model on assumed mode method. Artificial Neural Network control is used to track the desired trajectory accurately and get rid of vibrations (if any). The proceedings have been verified using Lyapunov formulations. Simulation results show that the method is very effective compared to other available control formulations in the literature.

C. Sun and J. Hong in [28] have mitigated vibrations on robotic manipulators and have based their control algorithm with Adaptive neural Networks. The system is modelled using lumped spring-mass approach and simulations are carried out. uniform ultimate boundedness of the closed-loop system is ensured so that the control algorithm is effective and output feedback control and full-state feedback control as well as are proposed individually.

H. C. Nho and P. Meckl in [29] proposed a new control architecture based on a simple proportional derivative (PD) encapsulated with fuzzy-neuro logic. The objective is to make a robotic manipulator robust to variable payloads. The neural network is trained by estimating masses using fuzzy logic and the inverse dynamics of the plant is ascertained. Experimental results show that the developed control logic is superior to most conventional controllers.

Yang Gao, Meng Joo Er and Song Yang in [30] have performed for multilink manipulators motion control using a neuro-fuzzy controller. The characteristic features of this controller are: (1) self-organizing fuzzy neural structure; (2) online learning of the robot dynamics; (3) fast convergence of tracking error; and (4) adaptive control.

2.3 LITERATURE REVIEW ON SLIDING MODE CONTROL

J. J. E. Slotine and S. S. Sastry in [31] obtained an algorithm of feedback-based control for obtaining perfect trajectory tracking in a class of non-linear, time-varying physical systems in the presence of external perturbations and parametric uncertainties. The algorithm uses integral continuous feedback-based control, resulting in the state trajectory to slide along a time-varying sliding manifold in the state space model of the system. This idealized control law has precise trajectory tracking; but there are some high frequency noise components in the state trajectory. To correct the high frequency components, it is followed that the continuous control algorithms may be used to approximate the otherwise discontinuous control law and derive perfect trajectory tracking within some accuracy bounds without generating undesirable high-frequency noise signals. The procedure is used to the control a two-link robotic manipulator handling variable and unknown weights in a flexible automated production system environment.

Bailey E, Arapostathis A. in [32] have proposed a simple and efficient sliding mode control structure to reduce the complexity of design for robot arms where the construction of individually stable discontinuous surfaces is not necessary. They have described the structure of the robotic arm dynamics and Lyapunov's second method of energy functions in order to establish a sliding surface directly on the intersection of the switching manifolds.

K. S. Yeung and Y. P. Chen in [33] were the frontrunners to develop a controller algorithm for robotic manipulators using the concept of variable-structure systems (VSS) to handle the set-point angle regulation problem. The limitations of Variable Structure Systems control with coherent dynamic coupling is mitigated for a group of structures with positive symmetrical inertia matrices. Parametric Uncertainties are also taken care of in this designed method, which is then easily extendable to manipulators having higher number of links. The phenomenon of chattering in such algorithms are also solved by introducing sliding sectors.

L. C. Fu and T. L. Liao in [34] integrated a controller taking in accounts of solely on the estimates of the unknown bounds on the parametric uncertainties and a Variable Structured Control (VSC) algorithm is obtained under the system structure refering consideration. It is derived that the outputs of the closed-loop system systematically follow given output commands irrespective of the uncertainties while the boundedness of all signals are maintained inside the loop. All the commands inside the loop are seen to be bounded for all time. To prove the superiority of the controller, the controller is subjected to the case of a 2 degree-of-freedom (DOF) manipulator system considering various variable payloads.

Chun-Yi Su and Tin-Pui Leung in [35] created a SMC structure considering an adaptive system, which is used to identify the unknown parametric bounds, for the command tracking control of a two degree of freedom robot manipulators. A SMC variable structure control algorithm with the parametric bound estimation is proposed. The major outcome of this control structure lies in the use of a characteristic matrix, called the regressor matrix, which makes it easy for segregating the unknown model properties from the robotic arm dynamics. Considering the upper bounds of such uncertainties which are projected by a easy adaptive scheme, the presented VSS controller ensured the system's stability. The robustness formulation shows that in sight of the parametric uncertainties, which are assumed to be fast-varying as well as unbounded, the closed-loop system can still be essentially controlled. Chattering is also mitigated by utilizing the boundary layer considerations.

Yury Stepanenko, Yong Caos and Chun-yi su in [36] have selected an equation as a hyperplane in the system's state-space model leading in a Proportional Derivative -type sliding surface. Two different types of the control structures are proposed: regular and adaptive. The first is very easy and can work with an uncertain dynamic model; the only info needed is a bound on one characteristic parameter. The latter results in an on-line estimation for this system bound. Both controllers are defend external disturbances and unmodelled dynamic effects.

C. -. Su and Y. Stepanenko in [37] have designed a controller algorithm based on a general form for a sliding surface. Two algorithm strategies for adaptive SMC are projected in this text for nonlinear robotic manipulators. It is derived that, without any beforehand info of the manipulator parameters, the controllers guarantee the existence of the sliding motion on the sliding surface, which can be a nonlinear as well as time-varying manifold. In the sliding mode, the desired closed loop performance can be obtained by selecting a suitable sliding manifold beforehand and an outline on selection has been proposed.

Arie Levantin in [38] has synthesized a control structure that sticks a non-linear system to a given manifold and mantains it within this manifold even when constraints are considered. It follows that the shift of the system from its predetermined boundaries (sliding accuracy) is proportionally related to the switching time delay in the manifold. A whole new category of sliding modes and laws are presented and the idea of order of the sliding mode is also projected. They show a bounded control continuously variant on time, with few discrete values only in the control action's derivative. It also follows that the sliding perfectness is empirically proportional to switching time delay's square.

G. Bartolini, E. Usai and A. Ferrara in [39] have presented a nobel result to the problem of chattering in systems with variable structure methodology. This solution is motivated by the original bang-bang optimally control theory, which is previously found and denoted in terms of a control structure by proposing a suitable auxiliary function considering a second-order unstable system with no velocity in its states. Then the utilisation of the control structure is extrapolated through suitable changes, to the class of nonlinear systems with parametric nuances of more ideal types. The control structure does not need the use of observers bases and differential inequalities and can be easily utilized by exploiting the commercial ingredients as peak detectors or other simplified methods.

M. Kemal Cihz and K. S. Narendra in [40] addresses the trajectory following control problem of two-link robotic systems with changing or unknown parameters. A new self-adaptive controller law is presented to correct the overall tracking index. This method uses a greater number of dynamic models of the plant in the control structure using an indirect adaptive controller. The energy input that is given to the joint motors is obtained at each and every instant using identifier model that best aligns with the manipulator dynamics. They also present the dynamical formulation of the control algorithm and the stability norms of the complete plant.

Kou-Cheng Hsu, Wei-Yen Wang and Ping-Zong Lin in [41] have obtained a new approach for a category of non-linear and uncertain systems having multiple inputs containing sector nonlinearities and dead zone non linearities. A SMC scheme is laid to obtain stabilizing laws for such uncertain nonlinear structures which ensure a global reaching requirement. They can also work in the systems without any external non-linearities at all. Moreover, they also ensure that the trajectories are asymptotically convergent on the sliding surface by using few illustrative examples to verify their claims.

M. Hamerlain, T. Youssef and M. Belhocine in [42] have carried out their research on chattering in sliding mode control which occurs due to discontinuous control structure. They have considered a normalized sliding-mode actuation that sways on the derivative of defining law instead of the actuator input itself thus decreasing chattering. They have verified their proceedings on a SCARA type manipulator arm with three degrees of freedom for a trajectory tracking case and concluded that the controller is essentially robust. Experimental findings show that their results are superior in terms of mitigating chattering.

Yuzheng Guo and Peng-Yung Woo in [43] have presented a corrective fuzzy SMC control structure for manipulator systems. A SISO and adaptive fuzzy system is used to calculate the control gain vector in a SMC control structure and it is synthesized using Lyapunov formulations. They have also mathematically proved its stability and its asymptotic convergence for set-point control and trajectory tracking. Their results show that there is substantially low chattering and the steady state errors are enormously mitigated and works better than the general form of sliding mode control.

J. Shi, H. Liu, and N. Bajcina in [44] proposes a simple controller having pole placement capability that uses computed-torque-like structure based on integral sliding mode. The obtained sliding mode controller is integrated with a disturbance estimator for reducing chattering using a semi-continuous rectifying action given by using an extra low-pass filter. The time constant of the estimator aligns the controller in between reduction in chattering and system robustness.

A comparative study is also presented against the existing literature to show its superiority.

S. Islam and X. P. Liu in [45] have proposed a multi valued model/control-dependent SMC method which is proposed to mitigate the level of uncertainties and decrease observer-based controller gains. They have split parameters of compact set into a lower number of smaller and simple subsets and a potential SMC related to each of these subsets is designed. To identify a candidate model, the derivative of the Lyapunov function is used as a resetting criterion which approximates the plant closely at each and every instant of time. The parameter calculation of traditional adaptive SMC law is reset into a model that is best in obtaining the system among a finite set of potential models which is then applied on a two degree of freedom robotic manipulator and its claims of better performance are verified.

Ouyang, Puren & Yue, Wen & Pano, Vangjel have in [46] utilised the advantages and the ease of ideal PD control and the high tracking performance of SMC and created a new hybrid PD-SMC strategy for trajectory tracking control of nonlinear robotic manipulators. The characteristic features of the proposed scheme are a model-free nonlinear feedback control and its globally asymptotical stability which are verified through numerical simulations under different operating conditions or tasks.

2.4 LITERATURE REVIEW ON ROBUSTNESS AND UNCERTAINTY

In [47] have projected a UDE (meaning uncertainness and perturbation Estimation) based controller for trajectory following problem. Non-linearities, external perturbation and system parametric uncertainties are nullified using the UDE and a FL (Feedback Linearization) based controller is integrated for trajectory tracking and it is realized by the UDE-estimated discrepancies to obtain robustness. An observer that considered the UDE-estimated uncertainties for robustness was highlighted because the resulting controller required joint velocities as well as joint positions, giving allowance to the UDE-based controller–observer system. The characteristic of this implementation is that it do not require any exact plant model. Lastly, the proposed design is verified using Quanser's single-link flexible joint module.

In [48] proposes a nobel approach to make a robust input–output linearization controller which is obtainded by calculating the unmeasurable nuances and external perturbations using a new uncertainty and perturbation estimator. A major highlight of the presented approach is that it does not need any knowledge about the nuances. The stability of the plant and the estimator is established.

J. E. Slotine and Li Weiping in [49] summarises the basic algorithms of adaptive controller for trajectory tracking problems of robotic manipulators. They assumed unknown dynamic parameters of the robotic manipulators and estimated that within a fraction of a second of a typical run, the system trajectory can be accurately and precisely controlled. The outcomes show that the level of robustness enjoys the same as unmodeled dynamics in a Proportional Derivative controller yet trajectory tracking properties are much better. Their approach is demonstrated on a high-speed 2 DOF quasi-direct-drive robot.

Zhong QC, Rees D In [50] have proposed a control strategy for uncertain Linear Time Invariant Systems dependent on an uncertainty and perturbation estimator (UDE).The performance-index is identical to time based delay control (TDC) but has advantages of no having any delay in the system, no unstability in the control signal and there is no need to measure the derivatives of the state vector. Then, the robust stability of LTI SISO systems is analysed, using simulation runs to show the usefullness of the UDE-based control as a comparison to TDC.

R. Sharma and M. Gopal in [51] have used Markov game formulation to create an adaptive fuzzy controller for robot manipulators. Bounded external disturbances and unknown parameter variations are very easily handled by The Markov game framework and makes it a robust controller. The authors have proposed fuzzy Markov games for adapting the FQL (fuzzy Q-learning) to a continuous-driven form of Markov based game theory using a controlling signal to adjust the end part of a fuzzy Markov game control structure online. The Markov game-adaptive fuzzy controller uses a simple fuzzy inference system (FIS) which is calculated to be efficient and obtains a quick control that does not require exact dynamics of the robotic system. The superiority of the proposed controller is compared against all the leading control methodologies in literature and applied on a 2-DOF SCARA robot manipulator and it turns out to outperform all other controllers in respect of neutralizing errors and control energy specifications over different ideal trajectories.

CHAPTER 3

MATHEMATICAL MODELLING OF A 2-LINK ROBOTIC MANIPULATOR

3.1 KINEMATIC ANALYSIS OF TWO-LINK ROBOTIC MANIPULATOR

3.1.1 FORWARD KINEMATICS

Kinematics refers to the end-effector position after execution of a movement in terms of joint angles and the geometry of the links and the process of transcending to this outcome is known as forward kinematics. A limb which is mechanical device, converts arm muscle length magnitudes and the angles of the joints to the positions of the hand. In Fig. 3.1, an illustration of the movement of the robotic manipulator in vertical direction is presented.

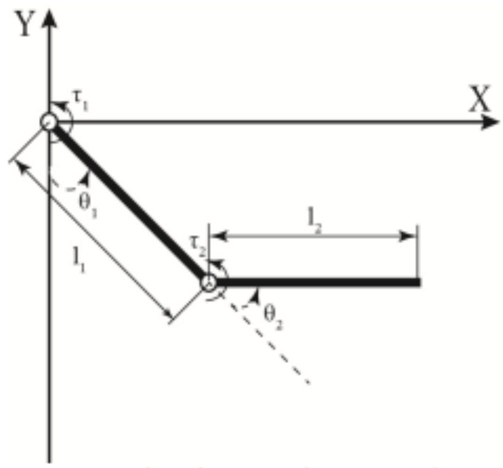


Fig 3.1 Illustration of robotic arm for forward kinematics

The arm is modelled in two solid links of length l_1 and l_2, the links turnabout horizontal and parallel axes which are fixed to the links. So, the forward kinematics from joint angles (θ_1, θ_2) to position of the hands (x, y) is given by

$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) \tag{3.1}$$

$$y = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) \tag{3.2}$$

3.1.2 INVERSE KINEMATICS

The transformation of the joint angles and muscle lengths from the respective desired hand position is termed as inverse kinematic transformation.

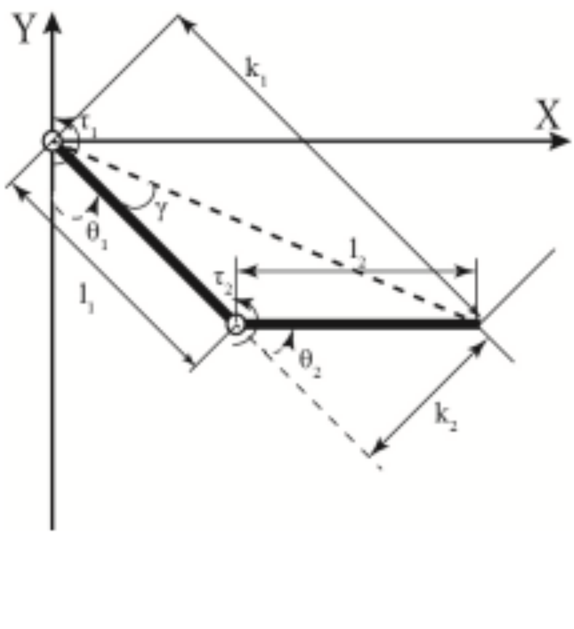


Fig 3.2 Illustration of robotic arm for inverse kinematics

So, mathematically the inverse kinematic translation for the generalized two link robotic manipulator from position (x, y) to angles (θ_1, θ_2) is given by

$$x^2 + y^2 = l_1^{\,2}\cos^2(\theta_1) + l_2^{\,2}\cos^2(\theta_1+\theta_2) + 2l_1 l_2\cos(\theta_1+\theta_2) +$$

$$l_1^{\,2}\sin^2(\theta_1) + l_2^{\,2}\sin^2(\theta_1+\theta_2) + 2l_1 l_2\sin(\theta_1)\sin(\theta_1+\theta_2) \quad (3.3)$$

The inverse dynamic can be implemented as an algorithm as follows

$$\theta_2 = a\tan 2(\sin(\theta_2), \cos(\theta_2))$$

$$= a\tan 2(\pm\sqrt{1-\cos^2(\theta_2)}, \cos(\theta_2))$$

$$= a\tan 2(\pm\sqrt{1-(\frac{x^2+y^2-l_1^{\,2}-l_2^{\,2}}{2l_1 l_2}}), \frac{x^2+y^2-l_1^{\,2}-l_2^{\,2}}{2l_1 l_2} \quad (3.4)$$

$$\theta_1 = a\tan 2(y, x) - a\tan 2(k_2, k_1) \quad (3.5)$$

$$k_1 = l_1 + l_2\cos(\theta_2) \quad (3.6)$$

$$k_2 = l_2\sin(\theta_2) \quad (3.7)$$

3.2 DYNAMIC ANALYSIS OF TWO-LINKROBOTIC MANIPULATOR

3.2.1 INTRODUCTION TO LAGRANGIAN FORMULATION

Lagrangian formulation is a reformulation of classical mechanics where the trajectory of a system of particles is obtained with equating the Lagrange equations where the system constraints are given special priority and are treated as separate equations, either by suing Lagrangian multipliers or by incorporating the constraints explicitly by appropriate choice of generalized coordinates. The system dynamics information is contained in functions called as Lagrangian function which is a form of the normalized co-ordinates with their corresponding time differentials.

The formulation is mathematically systematic although sophisticated. Application of Lagrangian formulation is ideal for mechanical systems where there are conservative forces and for neutralizing system constraint forces in any dimension whereas the actuated forces are considered by separating the driving forces to a sum of potential and kinetic forces, resulting in a class of improved Euler-Lagrangian representations. The idealized frames can be picked according to ease, to befit the symmetries in the structure or the geometry of the constraints, which may ease solving for the equation of motion of the system. The Lagrangian formulation is most popularly utilized to solve for problems in physics and when Newton's classical theory is not very much applicable. Lagrangian formulation is bestowed to the dynamics of particles, while fields are defined using a characteristic density function. Lagrange's methods also find application in optimization problems of dynamical structures.

Let us consider systems where there are only conservative forces. Such forces are obtained from a potential energy function $U(r_1, r_2 ... r_n)$ by differentiation as follows:

$$F_i = -\frac{\partial U}{\partial r_i} \tag{3.8}$$

Systems where the forces are purely conservative always conserve the total energy as such

$$E = K + U \tag{3.9}$$

Where K represents the Kinetic Energy and U represents the Potential Energy.

$$E = \frac{1}{2}\sum_{i=1}^{N} m_i \dot{r}_i^2 + U(r_1, r_2, ... r_n) \tag{3.10}$$

Differentiating the energy with respect to time:

$$\frac{dE}{dt} = \sum_{i=1}^{N} m_i \dot{r}_i . \bar{r}_i + \sum_{i=1}^{N} \frac{\partial U}{\partial r_i} . r_i \tag{3.11}$$

$$= \sum_{i=1}^{N} F_i \times \dot{r}_i - \sum_{i=1}^{N} \frac{\partial U}{\partial r_i} . \dot{r}_i$$

$$\Rightarrow \frac{dE}{dt} = O \tag{3.12}$$

Here, the law of conservation of energy is ensured where $\bar{r}_i = \frac{F_i}{m_i}$ (from Newtons Law of motion) and $F_i = -\frac{\partial U}{\partial r_i}$ (from definition of conservative force)

Thus, for conservative systems, we can transcend the above formulations into an elegant form of classical mechanics called as Lagrangian formulation. The Lagrangian function, L, for a system is described as the differential in energy between the kinetic and potential energy expressed as a function of positions and velocities. We can formulate it in an algorithm as

$$L(r,\dot{r}) = K - U \tag{3.13}$$

$$L = \sum_{i=1}^{N} \frac{1}{2} m_i \dot{r}_i^2 - U(r_1, r_2, \ldots r_n) \tag{3.14}$$

The classical equations of motion are then given by the Euler-Lagrangian formulation as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial r_i}\right) - \frac{\partial L}{\partial r_i} = 0 \tag{3.15}$$

The solution of the equations of motion for a given initial condition is known as a trajectory of the system.

3.2.2 DYNAMICS OF A TWO-LINK ROBOTIC MANIPULATOR USING EULER-LAGRANGIAN EQUATIONS

The equations of motion of a two-link robotic manipulator, shown in Fig. 3.3, is derived with the help of Euler–Lagrangian formalism as follows.

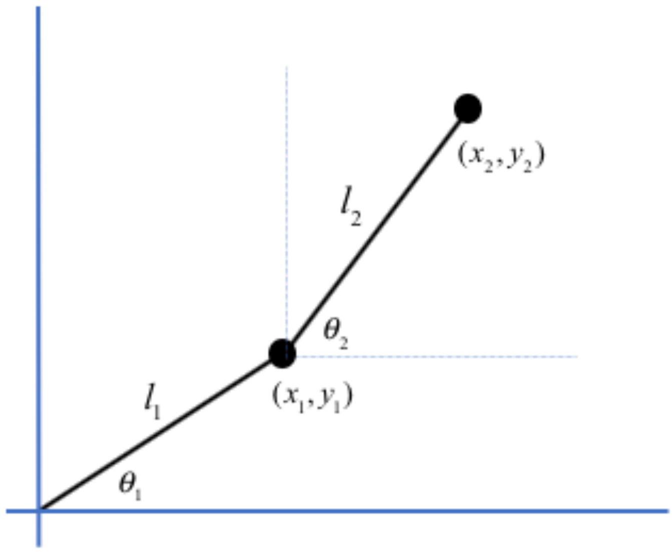


Fig. 3.3. Schematic diagram of a two-link robotic arm

L = Kinetic Energy – Potential Energy where L is the Lagrangian Formulation

$$f_{\theta_{1,2}} = \frac{d}{dt}[\frac{\partial L}{\partial \dot{\theta}_{1,2}}] - \frac{\partial L}{\partial \theta_{1,2}} \tag{3.16}$$

Here, Kinetic Energy, K.E $= \frac{1}{2}((m_1+m_2)l_1{}^2\dot{\theta}_1{}^2 + m_2 l_2{}^2(\dot{\theta}_1^2 + \dot{\theta}_2^2))$

$$+m_2 l_2 \dot{\theta}_1 (l_2 \dot{\theta}_2 + l_1 \cos(\theta_2))(\dot{\theta}_1 + \dot{\theta}_2) \qquad (3.17)$$

Potential energy,

$$\text{P.E} = g(m_1 l_1 \sin(\theta_1) + m_2(l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2))) \qquad (3.18)$$

$$L = KE - PE \qquad (3.19)$$

Solving the above equation, the equation of motion of the two-link robotic arm can be generalized as

$$M(\theta)\ddot{\theta} + C(\theta)\dot{\theta} + G(\theta)g = \tau_{in} \qquad (3.20)$$

Where $M(\theta)$ is the inertia matrix, $C(\theta)$ is the centripetal and Coriolis torque matrix, $G(\theta)$ is the gravity torque matrix and is the input torque vector. The various matrices are as follows

$$M(\theta) = \begin{pmatrix} (m_1 + m_2) l_1^2 + 2 l_1 l_2 m_2 \cos(\theta_2) & m_2 l_1 l_2 \cos(\theta_2) \\ m_2 l_1 l_2 \cos(\theta_2) + m_2 l_2^2 & m_2 l_2^2 \end{pmatrix}$$

$$\ddot{\theta} = \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix}$$

$$C(\theta) = \begin{pmatrix} -m_2 l_1 l_2 \sin(\theta_2)\dot{\theta}_2 & -m_2 l_1 l_2 \sin(\theta_2)(\dot{\theta}_1 + \dot{\theta}_2) \\ m_2 l_1 l_2 \sin(\theta_2)\dot{\theta}_1 & 0 \end{pmatrix}$$

$$\dot{\theta} = \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}$$

$$G(\theta) = \begin{pmatrix} (m_1 + m_2) l_1 \cos(\theta_1) + m_2 l_2 \cos(\theta_1 + \theta_2) \\ m_2 l_2 \cos(\theta_1 + \theta_2) \end{pmatrix}$$

$$\tau_{in} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

CHAPTER 4

SLIDING MODE CONTROL

4.1 INTRODUCTION TO VSCS AND SLIDING MODE CONTROL

The advent of Variable Structure Control Systems (VSCS) is credited to Emel'yanov and Barbashin of Russia in the early nineteenth century and ever since have been adopted from 1970s when a formal translation of this new control scheme was published in English. VSCS finds their application in robust regulators, model-reference adaptive systems, trajectory tracking systems, observer-based control structures and fault detecting schemes. The applications of VSCS have range from chemical processes to robotics, automatic flight controls, helicopter stability, space systems and much more. Sliding mode control is derived from VSCS and consists of isolated structures of characteristic properties calibrated by a switching logic. The system is said to be on the Sliding Phase when it reaches such defined structure. A surface $s(x)=0$ is traded attractive when trajectories starting in such surface, continue in it or others starting outside redirect to it. There occurs a discrete nature of the system trajectory when the state sways on either way in trying to align with the sliding surface. By definition, sliding mode control (SMC) is a supreme control algorithm which is inherently immune to system nuances either in the form of perturbations acting on the system or non-idealities within the system itself.

SMC is also considered as the first practical way out whenever there is a doubt about the accurate system dynamics. An inherently robust controller, SMC overcomes all bias within a simple control algorithm.

4.2 ILLUSTRATION OF SMC USING A DOUBLE INTEGRATOR

For the purpose of illustration consider the double integrator given by

$$\ddot{x}(t) = u(t) \tag{4.1}$$

Consider a feedback control law,

$$u(t) = -kx(t) \tag{4.2}$$

where k is a strictly positive scalar.

The way of analysing the resulting closed-loop motion is by means of a phase portrait, which is a plot of velocity vs position.

Substituting equation (4.2) in (4.1), and integrating we obtain,

$$\ddot{x}^2 + x^2 = c \tag{4.3}$$

Where c is the constant of integration which is strictly positive and a function of the initial conditions of the system.

More generally, a plot of $\dot{x}$ against x represents an ellipse whose characteristics depends on the initial conditions as shown in Figure 4.1. But the control law as defined in (4.2) is not appropriate because as shown in Figure 4.1, $\dot{x}$ and x do not tend to the origin as time tends to infinity.

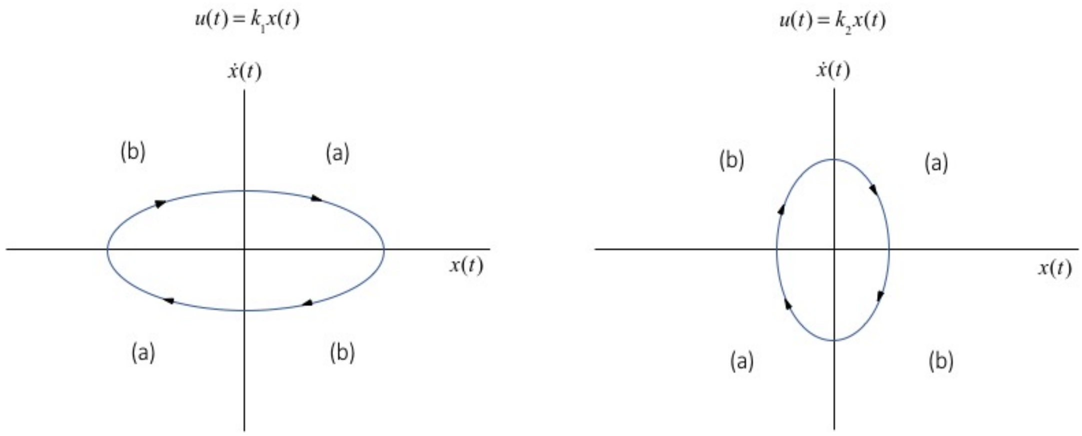


Figure 4.1: Phase portraits of Double Integrator

Consider a control law otherwise as,

$$u(t) = \begin{cases} -k_1 x(t), \rightarrow if \;\; x(t)\dot{x}(t) < 0 \\ k_2 x(t) \rightarrow otherwise \end{cases} \tag{4.4}$$

Where $0 < k_1 < 1 < k_2$. The phase plane is partitioned by four quadrants separated by axes as shown in Figure 1.1. The control law $u(t) = k_2 x(t)$ will be in effect for the phase plane in quadrant (a). In this region the distance of the points from the origin in the phase portrait decreases along the system trajectory. Similarly, in quadrant (b), when the control law is $u(t) = -k_1 x(t)$, the distance from the origin of the points in the phase portrait also decreases. The final phase portrait for the double integrator system is obtained by stitching together the respective regions from the two-phase portraits as shown in Figure 1.1. So, the resulting phase portrait is a spiral that narrows down towards the origin as shown in Figure 4.2.

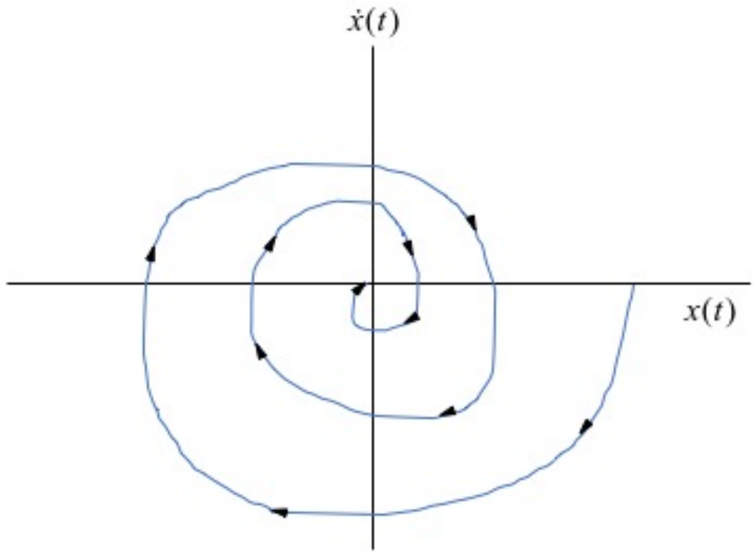


Figure 4.2: Phase portrait of system under VSCS

To verify the clauses of stability for this control action, we consider a energy function,

$$V\ (\dot{x}(t),\ x(t)) = \dot{x}^2(t) + x^2(t) \tag{4.5}$$

Which is otherwise the square of distance from the point ($\dot{x}(t), x(t)$) to the origin in the phase plane. The time derivative of $V(\dot{x}(t), x(t))$ of this closed loop system is given by,

$$\dot{V}(\dot{x}(t), x(t)) = 2\dot{x}(t)x(t) + 2\ddot{x}(t)\dot{x}(t) \tag{4.6}$$

$$= 2\dot{x}(t)(x(t) + u(t))$$

$$= 2x(t)\dot{x}(t)(1 - k_1) \rightarrow if\ \ \dot{x}(t)x(t) < 0$$
$$= 2x(t)\dot{x}(t)(1 - k_2) \rightarrow if\ \ \dot{x}(t)x(t) > 0$$

This is always made negative by tweaking the gain constants. Thus, the distance from the origin is always decreasing, which is the sole purpose of this control law. So, we introduce a rule for switching between the two control structures, which independently do not provide stability but provides a stable closed loop system.

4.3 CONTROL SCHEME FOR SMC

Consider a non-linear dynamical system described by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{4.7}$$

Where $x(t) \in R^n$ is an n-dimensional state vector and $u(t) \in R^m$ is a m-directional state feedback based input vector. A and B are functions $A : R^n x R \rightarrow R^n$ and $B : R^n x R \rightarrow R^{n \times m}$ are considered to be continuous functions and stable enough so that the Picard-Lindelof condition can ensure that a solution exists for the system and is singular.

The purpose of the feedback control law $u(x(t))$ is to bring the states of the system around the equilibrium point which is the origin. That is whatever deviation occurs in the states, the control law must ensure that the states are returned to the origin after the deviation. In SMC, the designer must ensure that the system behaves perfectly i.e. the system has a unique equilibrium considering that is limited to a subspace of its own configuration space. SMC controller ensures that the system trajectory is returned back to the subspace and is hold along that subspace so that the trajectories slide on it. The reduced order subspace is termed as the sliding manifold in the literature, and when the control law dictates trajectories to slid on it, it is directed to a sliding mode of system. Trajectories in line with this subspace is viewed as the eigen vectors of a Linear Time Invariant or LTI systems. Also, the sliding mode is associated with creation of a vector field with high-gain feedback. As an analogy we can think of a marble rolling along a groove where the groove is the sliding manifold and the mass of the marble or inertia is the gain of the SMC controller.

So, essentially SMC scheme includes:

1. Describing a reduced order structure such that the system undergoes the most favourable behaviour when the states of the system sticks to that manifold.
2. Obtaining feedback gains such that the states get attracted to the manifold and sticks to it.

The sliding mode design needs a switching function $\psi : R^n \to R^m$ that represents the distance of the state x from the desired trajectory.

- A state is on the sliding surface if $\psi(x(t)) = 0$
- A state is not on the sliding surface if $\psi(x(t)) \neq 0$

The SMC law takes into account of the sign of this distance and accordingly pushes or pulls in the line of the sliding surface where $\psi(x) = 0$. Suitable states will lead towards the sliding surface and since the control is discontinuous, the surface is established in finite time. When a state reaches the desired surface, it will creep on it and may move towards the origin where $\dot{x}(t)$ and $x(t)$ both are zero. So, the switching function ψ is like a contour map with a topography of constant width for trajectories to be forced upon.

The sliding manifold or the sliding surface is of dimension n x m where n is the number of states in $x(t)$ and m is the number of input signals in $u(t)$. For each index of control $1 \le k \le m$, there is a n x1 sliding surface which is described by

$$\{x(t) \in R^n : \psi_k(x(t)) = 0\} \tag{4.8}$$

To force the system states, satisfy $\psi(x(t)) = 0$, one must:

- Ensure the reachability of the system $\psi(x(t)) = 0$ from any initial condition
- When the reachability is ensured, the system is capable of keeping it at $\psi(x(t)) = 0$

4.4 THEORITICAL FOUNDATION OF SMC

<u>THEOREM 1</u>: The existence of the Sliding Mode

A candidate Lyapunov function checks the credibility of existence of sliding mode.

$$V(\psi(x(t))) = \frac{1}{2}\psi^T(x(t))\psi(x(t))$$

$$= \frac{1}{2}\left\|\psi(x(t))\right\|_2^2 \qquad (4.9)$$

Where $\|.\|$ is the Euclidean norm which is the distance from the sliding manifold where $\psi(x(t)) = 0$. For the system described by (4.7) and sliding surface defined by (4.8), the sufficient condition for the existence of sliding mode is that

$$\psi^T(x(t))\dot{\psi}(x(t)) < 0 \qquad (4.10)$$

In a neighbourhood of the surface given by $\psi(x(t)) = 0$.

In other words, the control law is picked so that $\psi(x(t))$ and $\dot{\psi}(x(t))$ have signs opposite to eachother because $u(x(t))$ has a direct impact on $\psi(x(t))$. That is,

- makes $\dot{\psi}(x(t))$ negative when $\psi(x(t))$ is positive.

- makes $\dot{\psi}(x(t))$ positive when $\psi(x(t))$ is negative.

Reachability: The sliding manifold is achieved in finite time

To ensure Reachability in finite time, must have less affinity for the origin that is if it does not vanish very easily, the resolution towards the sliding mode will not just be asymptotic but full proof. To make sure that

$$\dot{V}(\psi(x(t))) \leq -\mu(V(\psi(x(t))))^{\alpha} \tag{4.11}$$

Where $\mu > 0$ and $0 < \alpha \leq 1$ are constants.

The statement ensures that for the neighbourhood of the sliding mode $V(\psi(x(t))) \in [0,1]$

$$\dot{V}(\psi(x(t)))) \leq -\mu(V(\psi(x(t))))^{\alpha} \leq -\mu\sqrt{V(\psi(x(t)))} \tag{4.12}$$

So, for $V(\psi(x(t))) \in (0,1]$

$$\frac{1}{\sqrt{V(\psi(x(t)))}}\dot{V}(\psi(x(t))) \leq -\mu \tag{4.13}$$

which, by chai rule (i.e. $\frac{dW}{dt}$ with $W \triangleq 2\sqrt{V(\psi(x(t)))}$), means

$$D^{+}(2\sqrt{V(\psi(x(t)))}) = \frac{1}{\sqrt{V(\psi(x(t)))}}\dot{V}(\psi(x(t))) \leq -\mu \tag{4.14}$$

Where D^+ is the upper right hand derivative of $2\sqrt{V(\psi(x(t)))}$ and the symbol α represents proportionally. So by comparing to the curve $\dot{z} = -\mu$ with initial condition $z(0) = z_0$, it is the case that $2\sqrt{V(\psi(x(t)))} \leq V_0 - \mu t$ for all time.

Also because $\sqrt{V(\psi(x(t)))} > 0$, $\sqrt{V(\psi(x(t)))}$ must reach $\sqrt{V(\psi(x(t)))} = 0$ in finite time . Because $\sqrt{V(\psi(x(t)))}$ is directly proportional to the Euclidean norm of alternating function $\psi(x(t))$, this result shows that the rate of approach to the sliding mode must be strictly divergent.

For the case, when switching function $\psi(x(t))$ is scalar valued, the essential condition happens to be

$$\psi(x(t))\dot{\psi}(x(t)) \leq \mu\left|\psi(x(t))\right|^{\alpha} \tag{4.15}$$

Considering $\alpha = 1$, (4.15) reduces to

$$\psi(x(t))\dot{\psi}(x(t)) \leq \mu\left|\psi(x(t))\right| \tag{4.16}$$

That is $\operatorname{sgn}(\psi(x(t))) \neq \operatorname{sgn}(\dot{\psi}(x(t)))$ and

$$\left|\dot{\psi}(x(t))\right| > \mu > 0 \tag{4.17}$$

Meaning the system should always drift towards the switching surface $\psi(x(t)) = 0$ and its speed $\left|\dot{\psi}(x(t))\right|$ of moving towards the switching surface should have a lower bound which is non-zero. To make sure that, sliding mode controllers are not continuous around the $\psi(x(t))) = 0$, they switch their states from one the states pass through the manifold.

<u>THEOREM 2</u>: The Region of Attraction

The subspace for which the surfaces given in (1.8) is reachable is described by

$$\{x(t) \in R^n : \psi(x(t))\dot{\psi}(x(t)) < 0\} \tag{4.18}$$

Meaning that when the initial conditions come entirely from this space, the Lyapunov candidate function $V(\psi(x(t)))$ and states $x(t)$ are bound to get attracted to the sliding surface where $\psi(x(t)) = 0$. Moreover, if the reachability conditions from Theorem 1 are already satisfied, the system will enter the region where $\dot{V}(\psi(x(t)))$ is definitely away from zero in some finite time. Hence the sliding mode will be attracted in finite time.

<u>THEOREM 3</u>: The Sliding Motion

$\dfrac{d\psi(x)}{dx} B(x(t), u(t))$ be non-singular. So, we can ascertain that the controllability of a system is ensured meaning, the states will slide towards the sliding manifold. Then, once the sliding surface at $\psi(x)=0$ where is obtained, the system will be put on that sliding manifold. Along that sliding manifold $\dot{\psi}(x)=0$ is constant, we can put that

$$\dot{\psi}(x)=0 \tag{4.19}$$

If the state $x(t)$ is stable with respect to the differential equation in (4.19), then the state will slide along this sliding surface towards its equilibrium.

Hence the control law is designated as

$$\dot{\psi}(x(t))=0 \tag{4.20}$$

For the equivalent control law $u(x(t))$.

So,
$$\frac{d\psi(x)}{dx}(Ax(t)+Bu(t))=0 \tag{4.21}$$

And so, the equivalent control law becomes

$$u(t)=-\left(\frac{\partial\psi(x)B}{\partial x}\right)^{-1}\frac{\partial\psi(x)}{\partial x}A \tag{4.22}$$

Meaning the actual control law $u(x(t))$ may not be continuous but the quick switching action around the sliding manifold creates an impression that the system is acted upon by a continuous control.

So, the system dynamics can be written as

$$\dot{x}(t) = A - B\left(\frac{\partial\psi(x)B}{\partial x}\right)^{-1}\frac{\partial\psi(x)}{\partial x}A \tag{4.23}$$

$$A(I - B\left(\frac{\partial\psi(x)B}{\partial x}\right)^{-1}\frac{\partial\psi(x)}{\partial x}) \tag{4.24}$$

The resulting system comes in line with the sliding manifold differential equation as in (4.20) and the system trajectory equation reduce to the above equation (4.23) when coming from the reaching phase. Hence when the system comes down to the sliding manifold, it can be assumed to follow simpler conditions as given in (4.20). The same motion is approximately maintained when $\psi(x(t)) = 0$ holds true.

The inferences that follow from these theorems is that the sliding motion is insensitive to uncertainties and disturbances that are acting on the system. Also as long as the control effort is significant enough to ensure $\psi^T(x(t))\dot{\psi}(x(t)) < 0$ and $\dot{\psi}(x(t))$ is uniformly bounded away from zero, the system will be maintained on the sliding surface as if there is no disturbance. The invariance characteristics of this controller make it essentially robust.

4.5 SMC CONTROL ON THE ROBOTIC MANIPULATOR

For a single link, let θ_d be the desired or reference arm angle and θ be the actual arm angle at any particular instant. The error is defined as

$$e=\theta_d-\theta \tag{4.25}$$

Let a sliding surface be considered as

$$s=\dot{e}+\lambda e \tag{4.26}$$

whose time response is an exponentially decaying function as shown in Fig. 4.4 with time constant λ^{-1} driving the error asymptotically to 0 as $t\to\infty$, whenever $s=0$.

Now, we define a control effort,

$$u_{sw}=-\mathrm{sgn}(s) \tag{4.27}$$

Where
$$\mathrm{sgn}(s)=\begin{matrix} -1, s<0 \\ 0, s=0 \\ 1, s>0 \end{matrix} \tag{4.28}$$

which will actuate the system whenever $s\neq 0$.

The SMC design proceeding can be defined as creating a controller that makes sure the stabilization of s at 0 is obtained irrespective of any deviation in the dynamic model of the plant, and ensures that the state (here error) obeys the surface $s=0$ (as shown in Fig. 4.3)

For both the links, the error and the control effort can be generalized as vectors $e=\left[e_1 e_2\right]^T$ and $u_{sw}=\left[u_{sw1} u_{sw2}\right]^T$ respectively and each error term is dictated with some sliding surface as described by Eq. (4.26). Accordingly, we can have two unique values of λ.

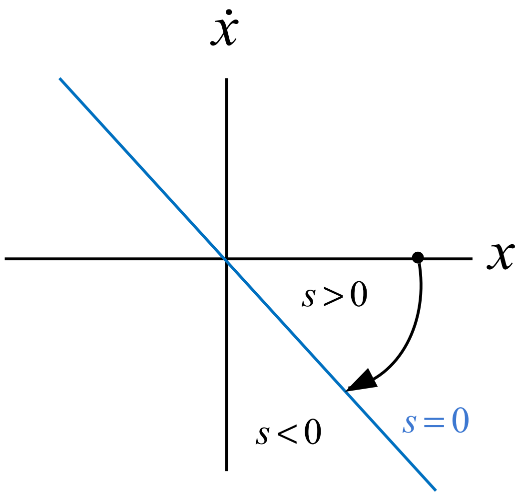


Fig. 4.3. Phase portrait of sliding for $s=0$

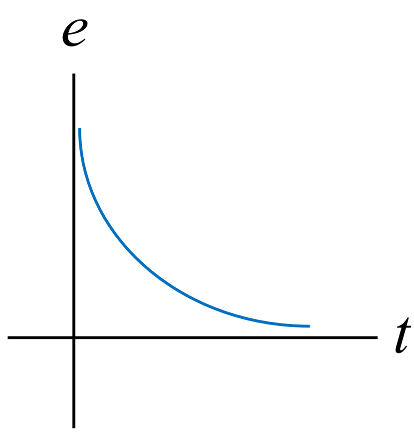


Fig. 4.4 Error dynamics surface

From (robot dynamics)

$$\ddot{\theta}=-M(\theta)^{-1}(C(\dot{\theta},\theta)+G(\theta))+M(\theta)^{-1}\tau_{in} \tag{4.29}$$

$$\ddot{\theta}=f(\theta)+u \tag{4.30}$$

where

$$f(\theta)=-M(\theta)^{-1}(C(\dot{\theta},\theta)+G(\theta)) \tag{4.31}$$

$$u = M(\theta)^{-1}\tau_{in} \tag{4.32}$$

$$\tau_{in} = M(\theta)u \tag{4.33}$$

From Eq. (4.26), putting $s = 0$, we have

$$\dot{e} + \lambda e = 0 \tag{4.34}$$

$$\ddot{e} + \lambda \dot{e} = 0 \tag{4.35}$$

From (4.25) and (4.35)

$$\ddot{\theta}_d - \ddot{\theta} + \lambda \dot{e} = 0 \tag{4.36}$$

From (4.30) and (4.36)

$$u_n = -f(\theta) + \ddot{\theta}_d + \lambda \dot{e} \tag{4.37}$$

Where u_n is the nominal control effort which can be represented as a vector $u_n = \left[u_{n1} u_{n2}\right]^T$.

Now, we have two control efforts that are actuating the system, one obtained from Eq. (4.37) which confirms the nominal dynamic equilibrium of the states of the system(here the state of interest being error) and the other, obtained from Eq. (4.27) which maintains the system at the required dynamic equilibrium whenever the states of the system (here the state of interest being error) gets deviated on account of any disturbances, uncertainties, unmodelled parameters and arbitrary initial conditions of the states.

So, the actuator effort can be described as a sum of two torques

$$\tau = \tau_n + \tau_s \tag{4.38}$$

Where

$$\tau_n = M(\theta)u \tag{4.39}$$

is obtained from Eq. (4.33) and Eq. (4.39) represents the nominal torque and

$$\tau_s = -K\,\mathrm{sgn}(s) \tag{4.40}$$

which is obtained from Eq. (4.27) represents the discontinuous or corrective torque that provides for any shift from the switching surface.

Both τ_n and τ_s are vectors represented by $\tau_n = \left[\tau_{n1}\,\tau_{n2}\right]^T$ and $\tau_s = \left[\tau_{s1}\,\tau_{s2}\right]^T$ respectively.

Here K is a diagonal controller discontinuous gain matrix.

4.6 CHATTERING IN SMC

The control law however has an undesirable phenomenon called chattering (as shown in Fig. 4.5), where momentum of the states or a minute uncertainty drives the sliding surface away from 0. These deviations are further intensified by the gain K in the control law which might lead to significant fluctuations in the controller command and hence the actuator outputs. To circumvent or come around chattering, the discontinuity of the SMC control law is smoothened (as shown in Fig. 4.6) out by bringing a boundary layer around the sliding surface within which the deviations from the sliding surface is ignored. Mathematically,

$$u_{sb} = -K \times sat(s) \tag{4.41}$$

Where

$$sat(s) = \begin{matrix} -1, & \frac{s}{\phi} > 1 \\ 0, & -1 < \left|\frac{s}{\phi}\right| < 1 \\ +1, & \frac{s}{\phi} < -1 \end{matrix} \tag{4.42}$$

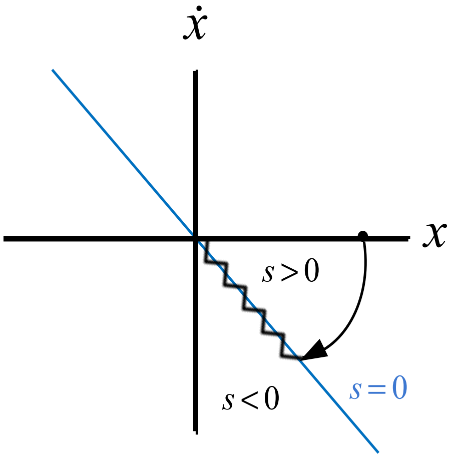


Fig. 4.5. Chattering in SMC

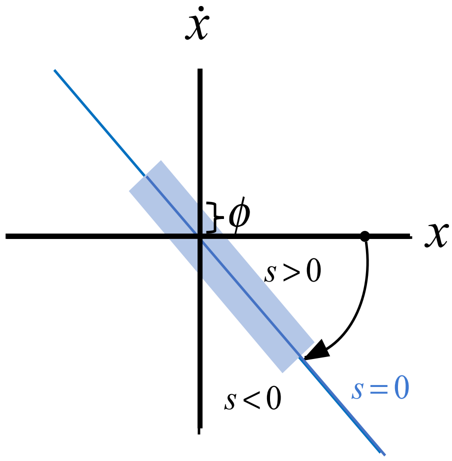


Fig. 4.6. SMC with boundary layer

The thickness or width of this boundary layer is denoted by ϕ.

CHAPTER 5

PARAMETRIC UNCERTAINTIES AND DISTURBANCES IN ROBOTIC MANIPULATORS

Robotic manipulators are always acted upon unpredictable dynamics say in the form of unpredictable gravitational torques, unmodelled friction torques, external disturbances, and inertia. Those uncertainties are tough to predict well before hand and acted upon. This results in degraded positioning as certainty and incomplete repeatability. So it is necessary to nullify or attenuate the disturbing nuances to achieve the best specifications. In most cases SMC or ASMC-based control methods can inherently make the system asymptotically stabilized in response to parametric uncertainties and disturbance estimations. But beyond such achievements, the operator may demand outstanding performance and call for trajectory tracking achievements in micro seconds with outmost accuracy. Few available methodologies of achieving such demands are

5.1 SLIDING PERTURBATION OBSERVER

The concept of Perturbation Estimation is introduced in Sliding Mode Control which outcomes in a procedure called Sliding Mode Control with Perturbation Estimation (SMCPE). It involves a Perturbation or Disturbance vector which combines the consequences of all the uncertainties and external perturbations together and this calculation constitutes a real time compensation mechanism against unknown parametric uncertainties. The precision of this calculation is the deciding consideration for robustness in this structure. As a result, the actuating terms of the error dynamics are minimised from the unknown uncertainties (as in the conventional Sliding Mode Control) to the precision within their calculations. This results in an improved command following accuracy without being more conservative stabilizing. SMCPE opens up an important design aspect which is design of disturbance observers for the controller. Perturbation observers should be precisely exact within the frequency range of consideration to make the system robust. But this methodology is expensive because of the cost of high quality sensor and full state feedback terms, which are most important to get efficient performance levels. But there is an issue with such structures which is the closed loop stability is very tough to achieve. The perturbation estimators which is obtained by applying numerical methods on the state feedback vector has also few constraints in the estimation phase itself because high efficiency filters are needed in applications with noisy velocity feedback. Another efficient state estimator that fits well for nonlinear and uncertain systems is Sliding observer (SO) which has a partial state feedback. Error in Estimation of the available output is the sliding function of this observer. Another observer in literature is Luenberger Observer, which is a basic SO structure consists of switching functions added to a traditional SO. The result of perturbation estimation and SO is termed as Sliding Perturbation Observer (SPO) which creates a highly efficient algorithm immune to perturbations, utilizes only quasi state feedback is far better than traditional SMC

5.2 TIME DELAY CONTROL (TDC)

Time Delay Control does not depend on information of the system dynamics, constant iterative tasks, high switching frequencies or discontinuous control. The methodology requires direct estimate of derivatives of the states to estimate current prospects of unknown dynamics and perturbations linked with the system through time delay. The controller thus discussed uses the accumulated information to cancel the unnecessary dynamics and unknown disturbances simultaneously which is then nullified by considering necessary offset in the dynamics of the plant. The TDC bases earlier calculations of the response of the system and control inputs to immediately alter the control actions rather than fixing the vector values. The TDC algorithm has been applied to several non-linear systems as well as several real time setups which shows that the outcomes are always better in terms of effectiveness befitting way in line with expectations in performance.

5.3 MATHEMATICAL FORMULATIONS FOR A GENERALIZED UNCERTAINTY ESTIMATOR

The dynamics of a robotic manipulator is given by Eq (3.20)

$$\tau = M(\theta)\ddot{\theta} + C(\theta)\dot{\theta} + G(\theta)g$$

The above equation are linearly parameterizable as

$$M(\theta)\ddot{\theta} + C(\theta)\dot{\theta} + G(\theta)g = Y(\theta,\dot{\theta},\ddot{\theta})\phi \tag{5.1}$$

where ϕ is a constant px1 vector of robotic parameters and Y is a nxp matrix having known functions of the generalized coordinates and their higher derivatives.

The bound on the parametric uncertainty is considered as

$$\left\|\tilde{\phi}\right\| = \left\|\phi_0 - \phi\right\| \tag{5.2}$$

Where $\phi_0 \in R^n$ and $\rho \in R_+$. Since ρ is assumed to be unknown, ρ is estimated with the estimation law. The estimate of ρ is defined as

$$\tilde{\rho}(t) \triangleq \rho - \tilde{\rho}(t) \tag{5.3}$$

A nominal control vector τ_0 is hence defined as

$$\tau_0 = M(\theta)a + C(\theta)v + G_0(\theta) - Ks \tag{5.4}$$

Where $v = \dot{q}_d - \Lambda\tilde{q}$, $a = \dot{v}$, $s = \dot{\tilde{q}} + \Lambda\tilde{q}$ and $\tilde{q} = q - q_d$

q_d is given double continually differentiable reference trajectory and the gain matrices K and Λ are positive definite and diagonal matrices.

So there is a control input τ in terms of nominal control vector τ_0 as

$$\tau = \tau_0 + Y(\theta, \dot{\theta}, v, a) p(t)$$

$$= Y(\theta, \dot{\theta}, v, a)(\theta_0 + p(t)) - Ks \qquad (5.5)$$

Where $p(t)$ is the aditional effort to make the system immune to parametric uncertainties denoted by $\tilde{\phi}$.

So the overall control law becomes

$$M(\theta)\dot{s} + C(\theta, \dot{\theta})s + Ks = Y(\theta, \dot{\theta}, v, a)(\tilde{\phi} + p(t)) \qquad (5.6)$$

CHAPTER 6

SIMULATIONS AND RESULTS

In this chapter, we discuss the results obtained from simulating the various objectives of this research.

6.1 SIMULATION MODELS IN SIMULINK AND MATLAB

6.1.1 MODEL OF 2-DOF ROBOTIC MANIPULATOR

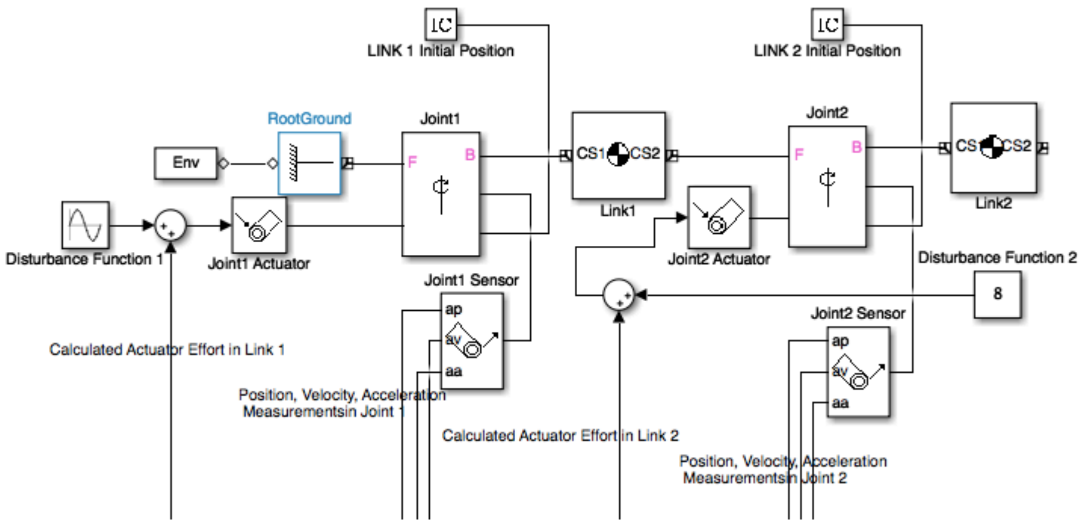


Fig. 6.1 Simulink Model for 2-DOF Robotic Manipulator

In this SIMULINK model, a 2-DOF robotic manipulator is formulated on which the proposed control architecture is based.

The various blocks utilised are

- MACHINE ENVIRONMENT – Denotes the mechanical simulation environment for the computer to which the block is connected: gravity forces, dimensions, analysis mode, constraint solver type, tolerances, linearizability, and visualization. And Visualization is activated in this block to get the real time simulation

  The gravity vector considered here is [0 -9.8 0]

- GROUND - Terminates one side of a Joint to a reference location in the generalized coordinate system.
  Here ground is at [0 0 0].

- REVOLUTE – Represents rotation with 1 Degree of Freedom. The follower (F) is allowed to rotate around base (B) and has a single axis of rotation. Sensor and actuator ports can be cascaded. Right Rand rule suggests the direction of forward motion provided the direction of axis and sequence of base follower.
  Number of sensors is 3 and the axis of action is [0 0 -1]

- BODY – Allows to describe a rigid mechanical body in terms of mass m, inertia J, and coordinate origins and axes for centre of gravity (CG) or other user-defined Body coordinate systems. By default, the origin and orientation has no initial values unless Body and/or connected Joints are described separately. This block has additional settins for customizable body geometry and colour.

As considered from Table 6.1, the mass is considered 0.7 kg and 0.5 kg for Link 1 and Link 2 respectively and the corresponding link lengths are 1 metre and 0.8 metres respectively.

- JOINT INITIAL CONDITION - Sets the initial linear/angular position and velocity of some or all of the primitives in a Joint. Connect to a Joint to see a list of its primitives.

  The default initial angles for both the links are 0 degrees.

- JOINT SENSORS - Measures linear/angular position, velocity, acceleration, computed force/torque and/or reaction force/torque of a Joint primitive. Spherical measured by quaternion. Base-follower sequence and joint axis determine sign of forward motion. Outputs are Simulink signals. Multiple output signals can be bundled into one signal. Connect to Joint block to see Connected to primitive list.

  Angle, Angular Velocity and Angular Acceleration are checked for this block.

- JOINT ACTUATOR - Actuates a Joint primitive with generalized force/torque or linear/angular position, velocity, and acceleration motion signals. Base-follower sequence and joint axis determines sign of forward motion. Inputs are

Simulink signals. Motion input signals must be bundled into one signal. Connect to Joint block to see Connected to primitive list. The default unit considered here is N-m.

- <u>DISTURBANCE FUNCTIONS</u> – The disturbance functions are sinusoidal and constant functions.

6.1.2 THE SLIDING MODE CONTROL ARCHITECTURE

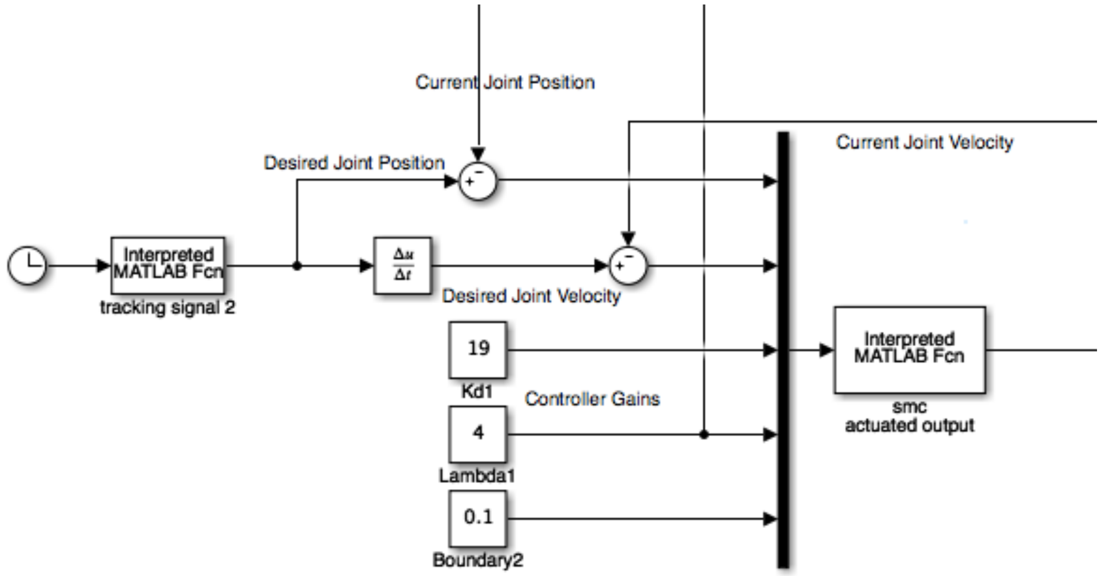


Fig. 6.2 Simulink Model for SMC Architecture

The output of the integrated MATLAB Fcn ‘smc actuated output’ actuates the system when the plant deviates from its ideality. This function along with the nominal dynamic torque drives the plant together.

The integrated MATLAB Fcn is called upon from a m-file contain the following script which corroborates with the equations governed in 4.25 to 4.40

The default values for the controller are as described in Table 6.2

```
function output = smc(e, e_dot, Kd, lambda, psi)
    s = lambda * e + e_dot;

    if (abs(s) >psi)
        sat_s = sign(s);
    else
        sat_s = (s/psi);
    end

    output = -Kd * sat_s;

     end
```

The integrated MATLAB Fcn ‘tracking signal’ identifies the desired trajectory that is applied to the links. In this case as in Fig. 6.2, the trajectory tracking command in Link 2 is shown.

6.1.3 SIMULATION MODEL FOR THE NOMINAL DYNAMIC MODEL OF THE TWO-DOF ROBOTIC MANIPULATOR

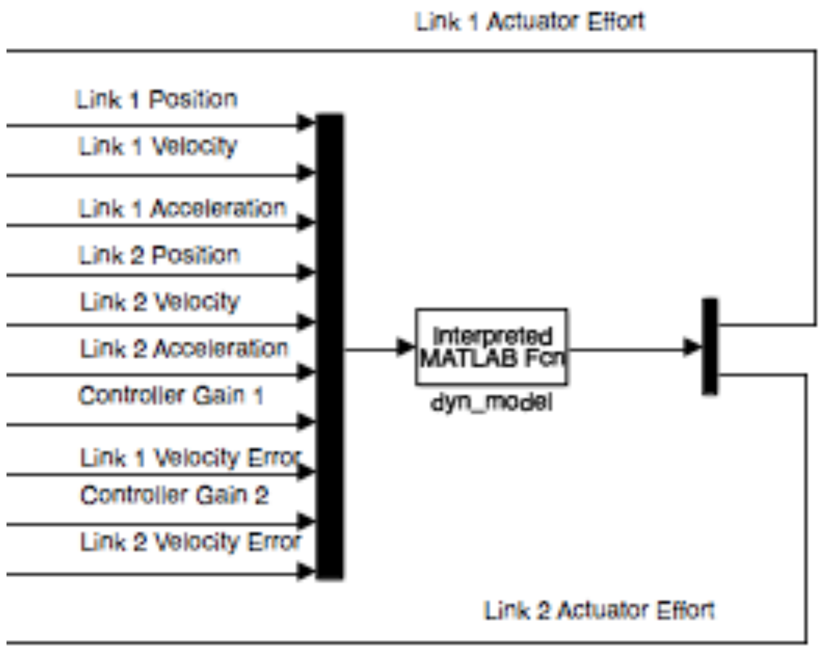


Fig. 6.3 Simulink Model for Nominal Dynamics of the plant

The integrated MATLAB Fcn 'dyn_model' superimposes the nominal dynamics of the Two-DOF robotic manipulator is called upon from a m-file contain the following script which corroborates with the equations governed in 4.25 to 4.37

```
function output = dyna(theta1, theta_dot1,
theta_dot_dot1, theta2, theta_dot2, theta_dot_dot2,
lambda1, error_dot1, lambda2, error_dot2)
    r1 = 1;
    r2 = 0.8;
    m1 = 0.7;
    m2 = 0.5;
    g = 9.8;

    M11 = (m1 + m2) * r1^2 + m2 * r2^2 + 2 * m2 * r1 * r2
* cos(theta2);
    M12 = m2 * r2 ^2 + m2 * r1 * r2 * cos(theta2);
    M22 = m2 * r2 ^2;
```

```
    F12 = m2 * r1 * r2 * sin(theta2);

    g1 = (m1 + m2) * r1 * cos(theta2) + m2 * r2 *
cos(theta1 + theta2);
    g2 = m2 * r2 * cos(theta1 + theta2);

        f_est1   =   M11   *   theta_dot_dot1   +   M12   *
     theta_dot_dot2  - F12 *  theta_dot2 * theta_dot1 - F12
     * (theta_dot1 + theta_dot2) * theta_dot2 + g1 * g;
         f_est2 = M12 * theta_dot_dot1 + M22 *
         theta_dot_dot2 +  F12 * theta_dot1 * theta_dot1
         + g2 * g;


    output(1,1) = -f_est1 + theta_dot_dot1 + lambda1 *
(error_dot1);
    output(2,1) = -f_est2 + theta_dot_dot2 + lambda2 *
(error_dot2);

    end
```

6.1.4 SIMULATION MODEL FOR THE OVERALL SYSTEM

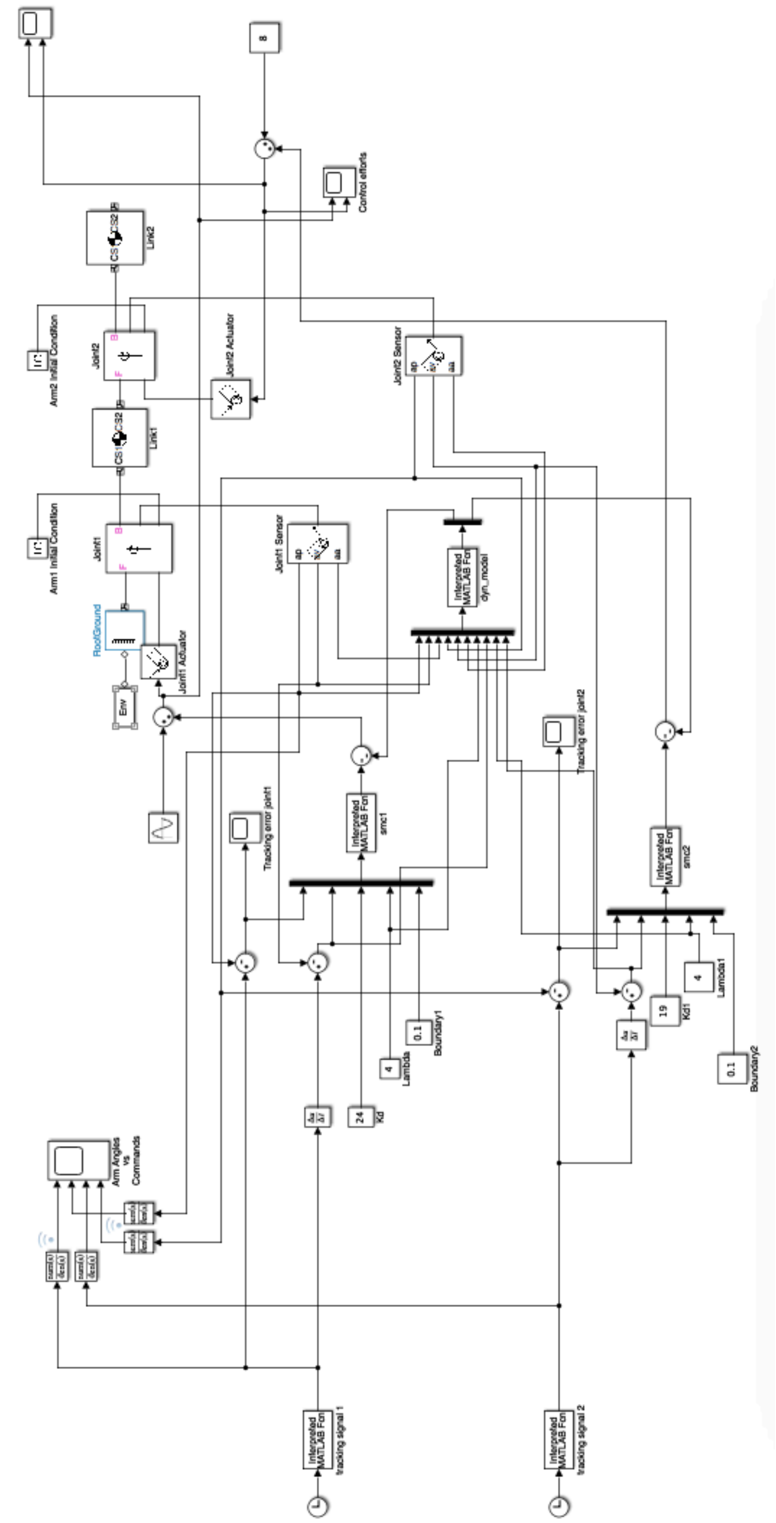


Fig. 6.4 Simulink Model for the overall system

6.2 SYSTEM SPECIFICATIONS

TABLE 6.1

SYSTEM PARAMETERS

Parameter	***Link 1***	***Link 2***
Mass	0.7 kg	0.5 kg
Length	1 meter	0.8 meter

TABLE 6.2

CONTROLLER PARAMETERS

Parameter	***Link 1***	***Link 2***
Gain matrix, K	$\begin{pmatrix} 24 & 0 \\ 0 & 19 \end{pmatrix}$	
Constant λ	4	4
Boundary width (ϕ)	0.1	0.1

TABLE 6.3

NATURE OF DISTURBANCE

Set	***Link1***	***Link 2***	***Figure***
Set 1	Sinusoidal $8\sin(t)$	Constant 8	11-13
Set 2	Sinusoidal $10\sin(t)$	Sinusoidal $-10\sin(2*t)$	14-16
Set 3	Random $rand(-10,10)$ $T_s = 0.5s$	Random $rand(-10,10)$ $T_s = 0.5s$	17-19

6.3 SIMULATION RESULTS

6.3.1 TRAJECTORY TRACKING TESTS

6.3.1.1 SLOW VARYING TRAJECTORIES

The links are subjected to the following tracking commands with both the arms initially at 0 degrees.

$$\theta_1 = 11.45 + 17.19\ (\cos(t)) \text{ degrees} \tag{6.1}$$

$$\theta_2 = 11.45 - 22.91\ (\sin(t)) \text{ degrees} \tag{6.2}$$

The response as obtained in Fig. 6.5 shows that the links follow the commands with outmost accuracy. Fig. 6.6 shows the corresponding actuator effort in terms of torque that needs to be subjected at the two links. The waveforms are obtained with lower values of controller gains because the system is not under any stress in the form of fast acting signals or disturbances. The gain values being $K_1 = 17$ and $K_2 = 15$. Also, with lower value of gains, the actuator effort required is comparatively lower. The waveforms show substantial chattering for which a high frequency actuating effort is required. This issue is mitigated using the boundary layer in the Sliding Mode Controller with boundary values as mentioned in Table 6.2. The corresponding actuator efforts obtained are shown in Fig. 6.7.

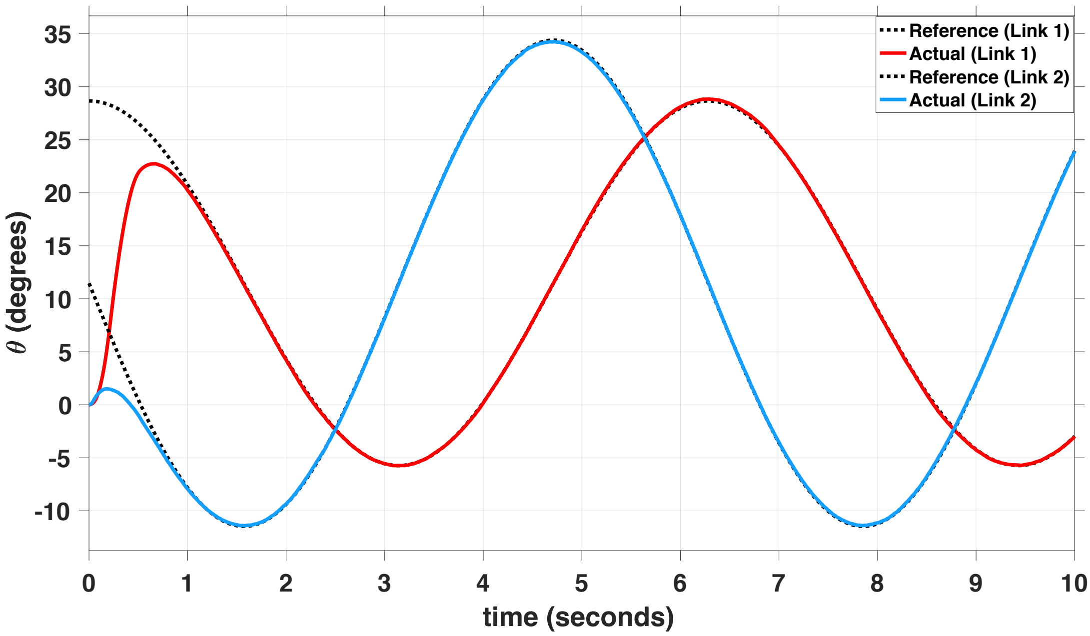


Fig. 6.5. Response of θ_1 and θ_2 with respect to slow varying tracking commands

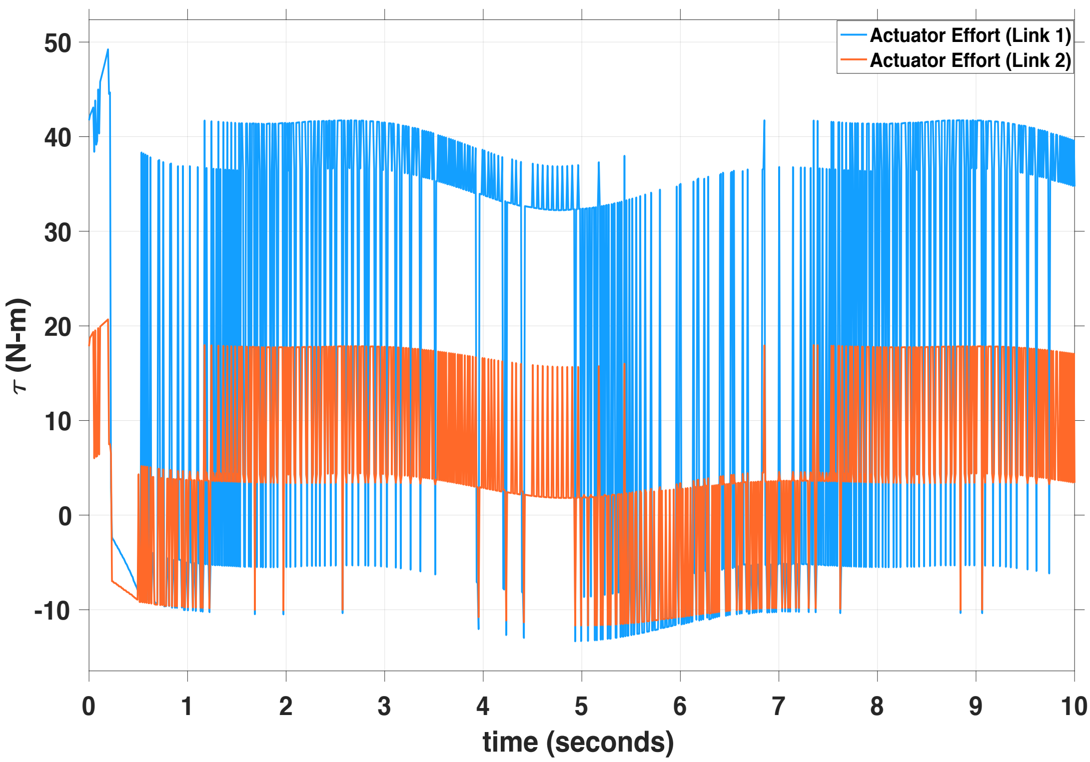


Fig. 6.6. Actuator efforts τ_1 and τ_2 for slow varying tracking commands

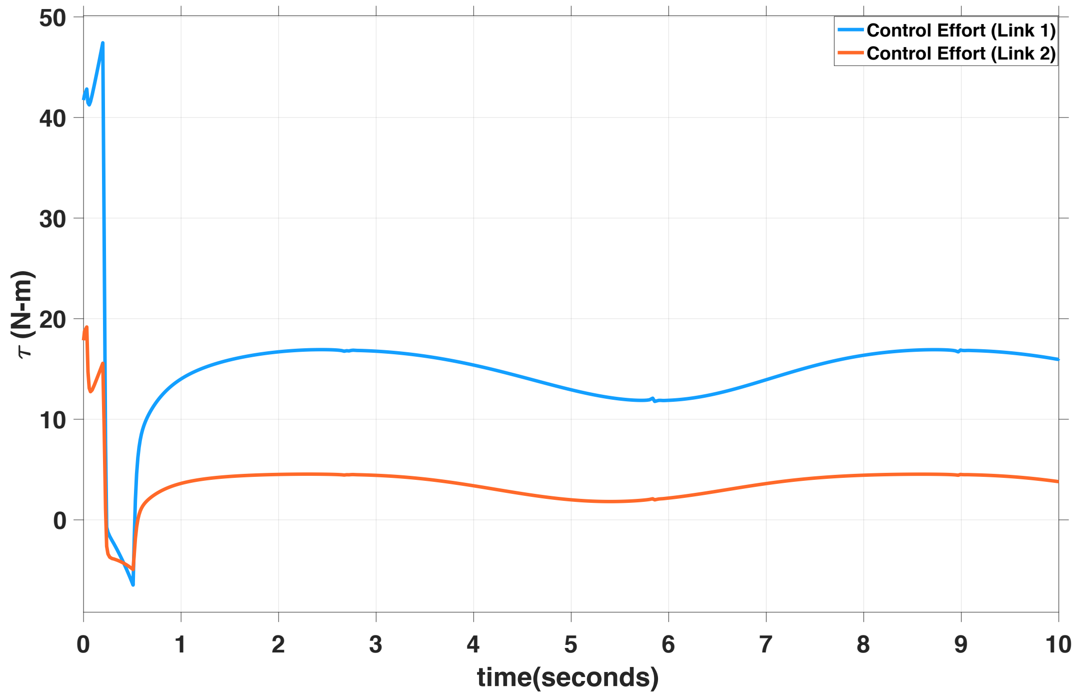


Fig. 6.7. Actuator efforts τ_1 and τ_2 for slow varying tracking commands with boundary layer

6.3.1.2 FAST VARYING TRAJECTORIES

Here, to check the trajectory tracking capability, the commands are applied with components having twice the angular frequency so as to understand how the controller can handle fast varying commands as in [51]. The arms are initially kept at $\theta_{10} = +20$ degrees and $\theta_{2o} = -20$ degrees and the links are subject to the following trajectories

$$\theta_1 = 11.45 + 17.19\ (\cos(t) + \cos\ (2*t)) \tag{6.3}$$

$$\theta_2 = 11.45 - 22.91\ (\sin(t) +\ \sin(2*t)) \tag{6.4}$$

The response of both the links are shown in Fig. 6.8 and the actuator torques are shown in Fig. 6.9

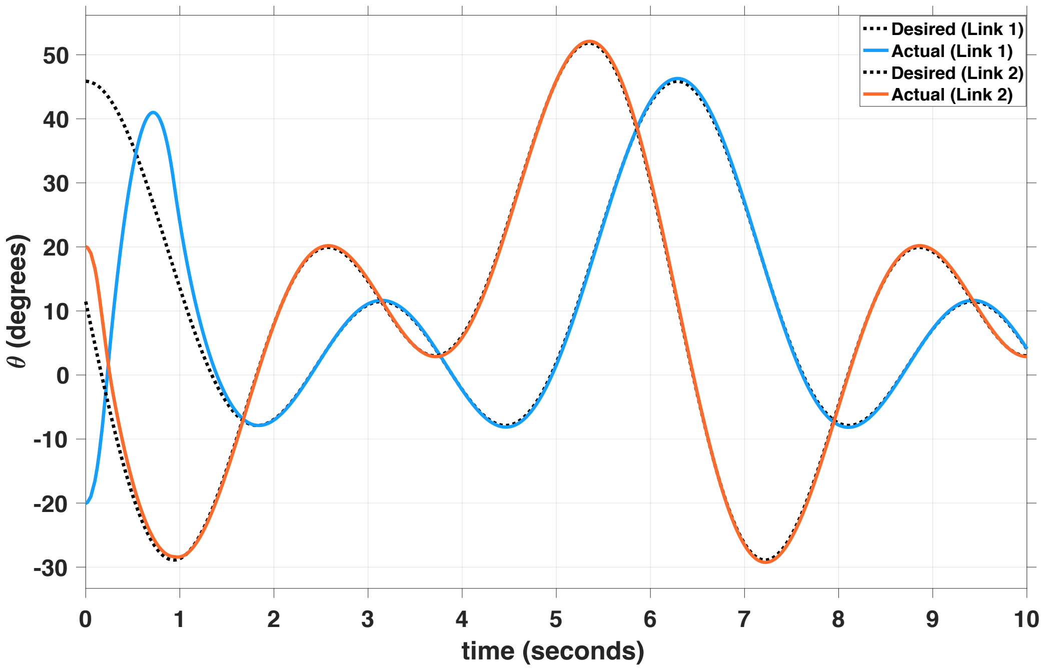


Fig. 6.8. Response of θ_1 and θ_2 with respect to fast varying tracking commands

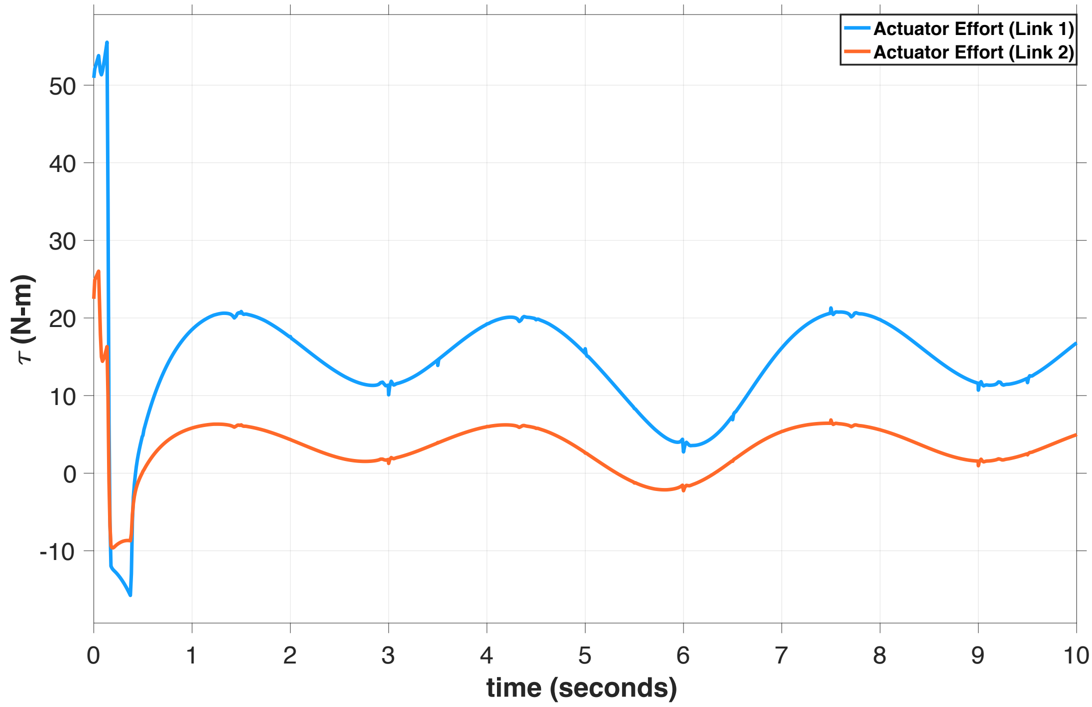


Fig. 6.9. Actuator efforts τ_1 and τ_2 for fast varying tracking commands with boundary layer

6.3.2 DISTURBANCE TOLLERANE TESTS

The system is now subjected to the commands as described by Eq. (6.1) and Eq. (6.2) and the following combinations of disturbances in terms of torque (N-m) are subjected to the links. These disturbances might be on account of a gust of wind, or varying dynamics of the load. The responses of the links along with the actuating efforts required are shown in figures as mentioned in Table 6.3.

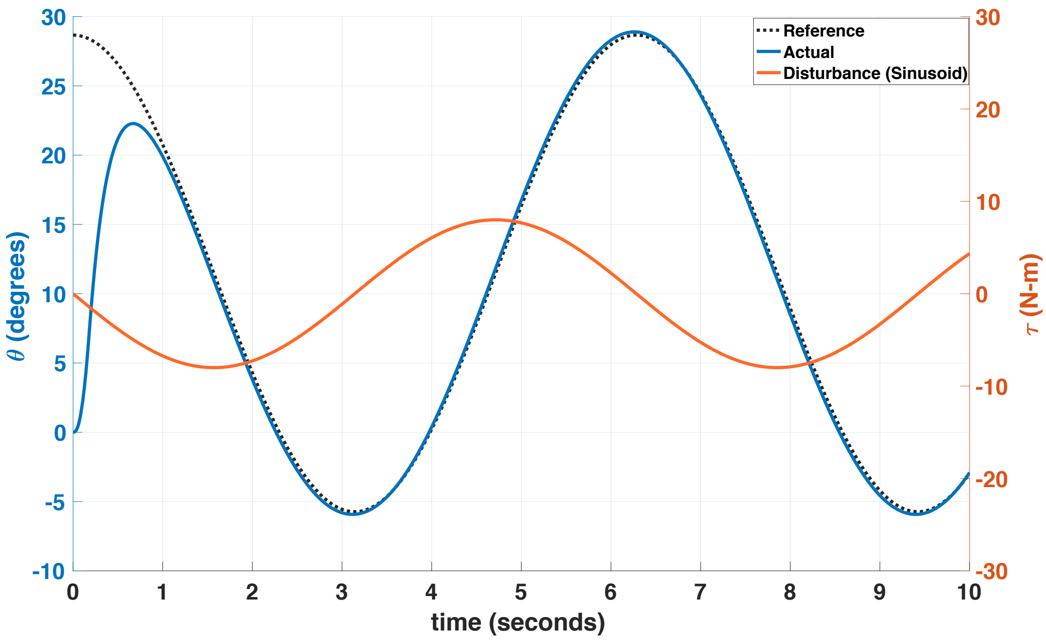


Fig. 6.10. Trajectory tracking of θ_1 with sinusoidal disturbance in link 1 and constant disturbance in link 2

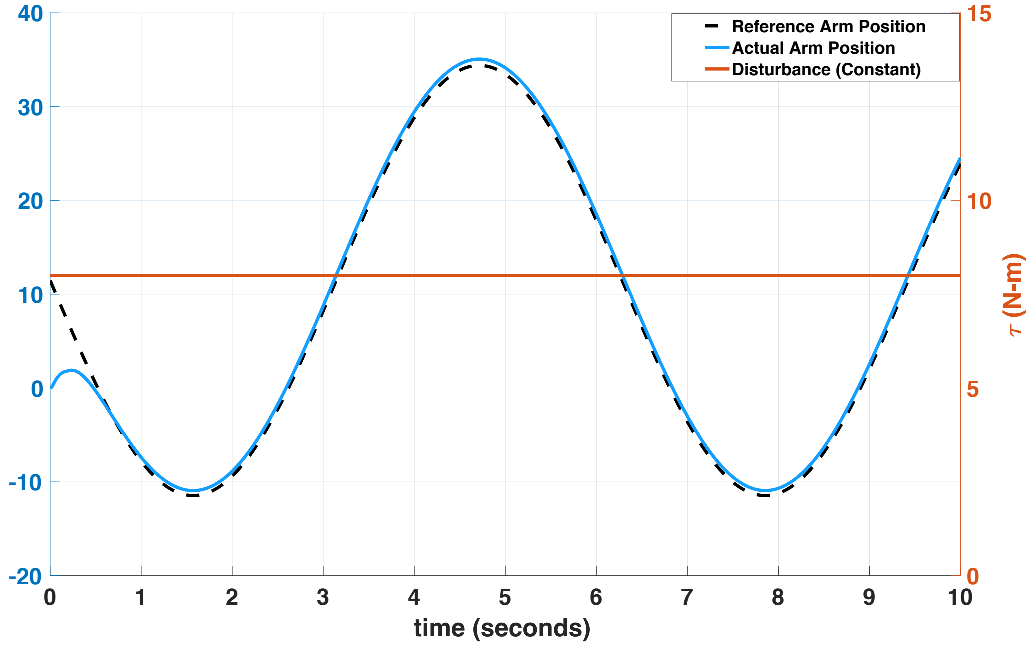


Fig. 6.11 Trajectory tracking of θ_2 with sinusoidal disturbance in link 1 and constant disturbance in link 2

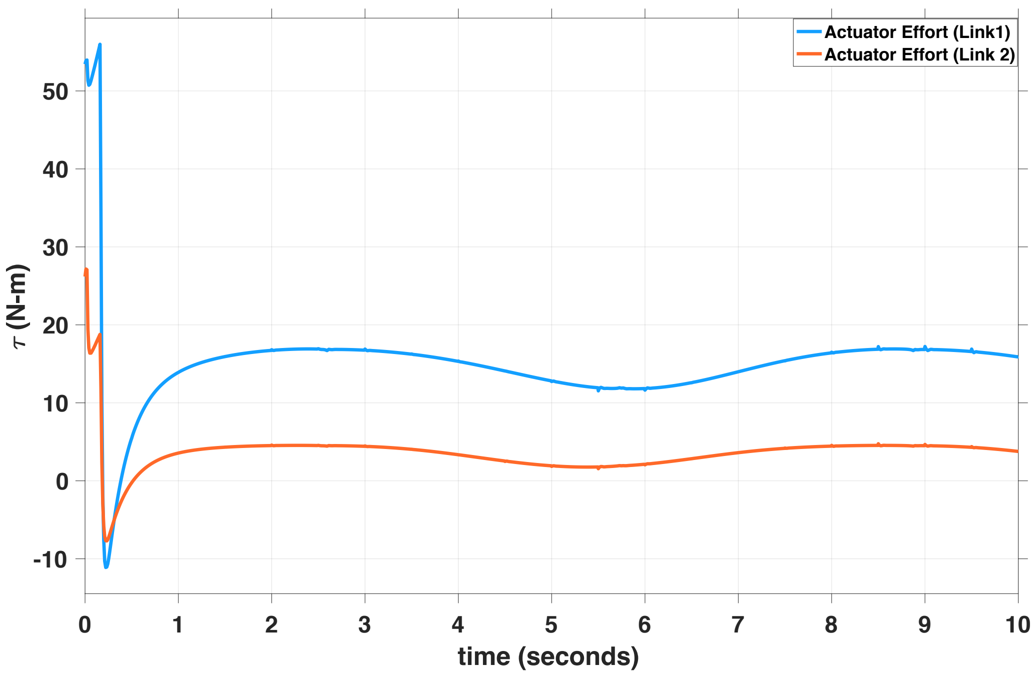


Fig. 6.12. τ_1 and τ_2 with sinusoidal and constant disturbances in link 1 and 2 respectively

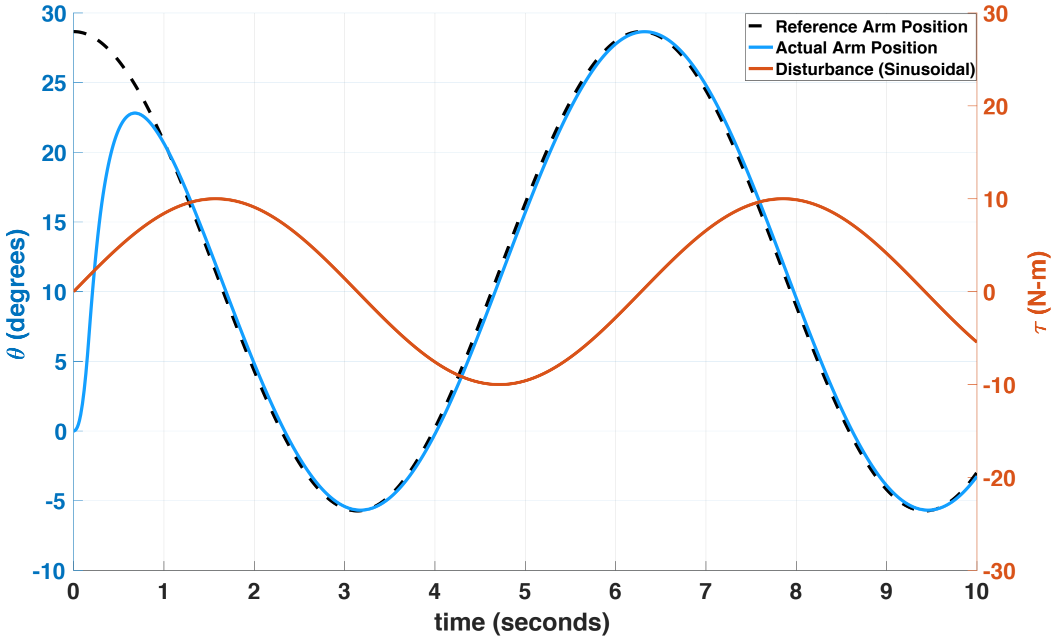


Fig. 6.13. Trajectory tracking of θ_1 with sinusoidal disturbances in both links

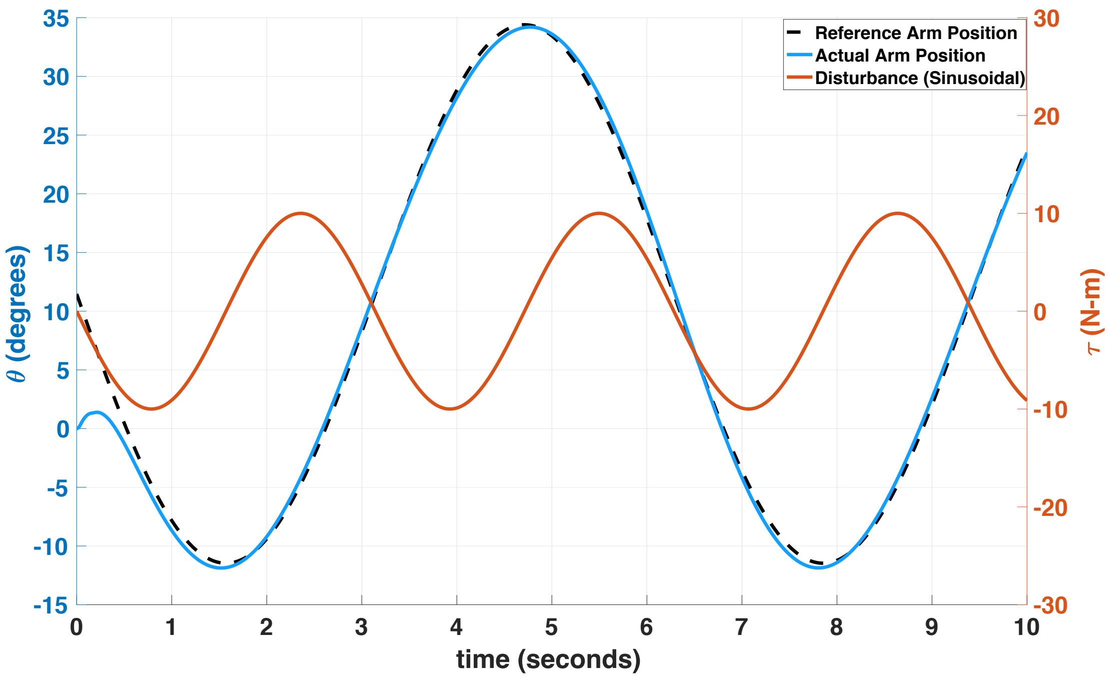


Fig. 6.14. Trajectory tracking of θ_2 with sinusoidal disturbances in both links

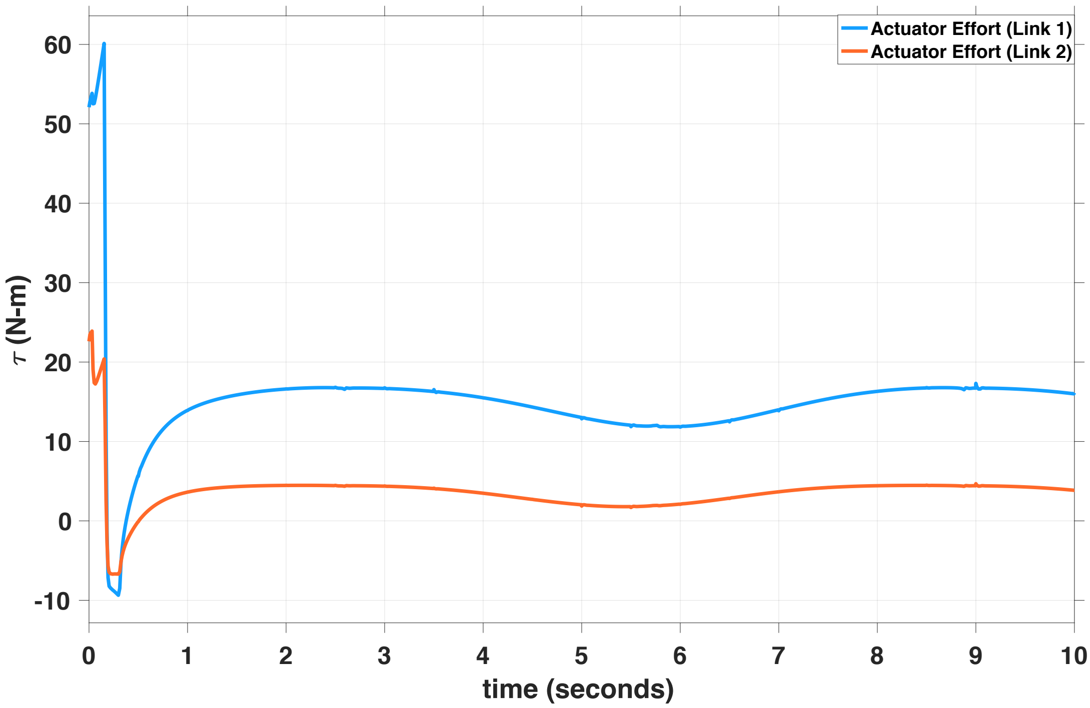


Fig. 6.15. τ_1 and τ_2 with sinusoidal disturbances in both links

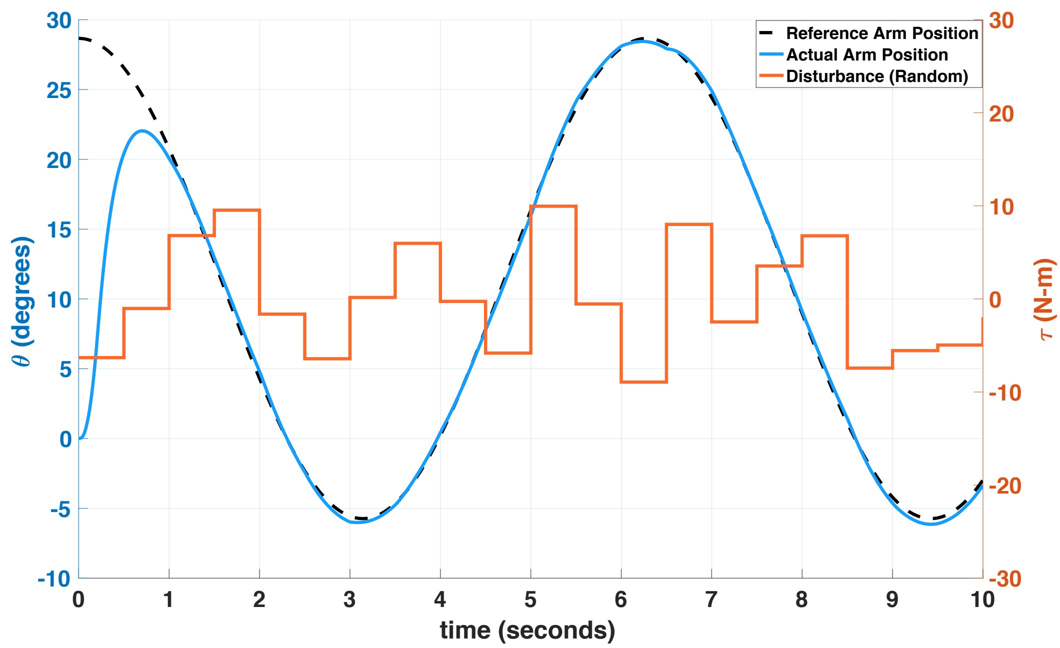


Fig. 6.16. Trajectory tracking of θ_1 with random disturbances in both links

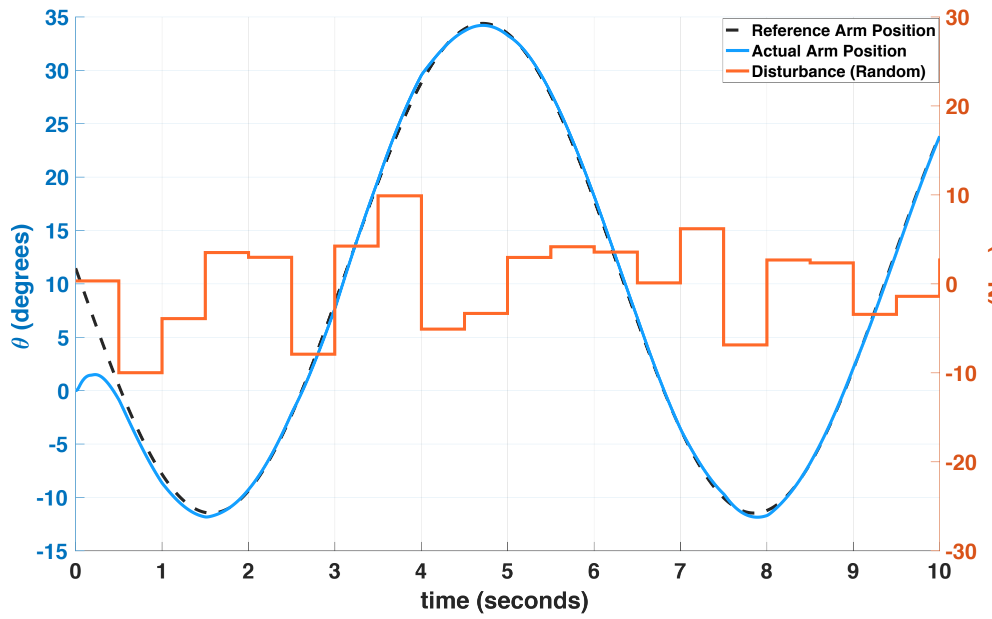


Fig. 6.17. Trajectory Tracking of θ_2 with random disturbances in both links

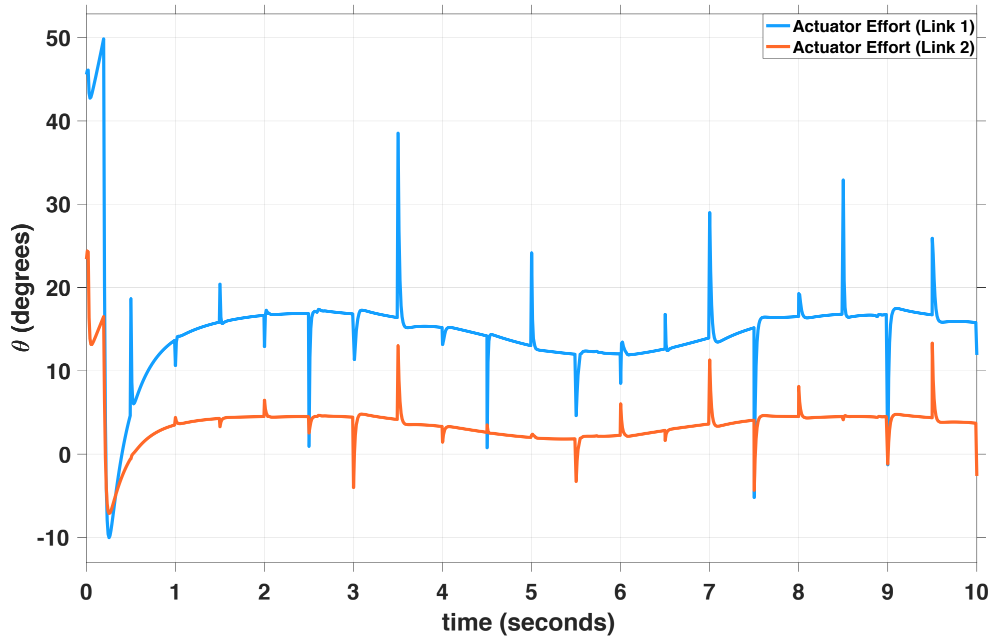


Fig. 6.18. τ_1 and τ_2 with random disturbances in both links

6.3.3 PARAMETRIC UNCERTAINTY TESTS

The robotic arm may be subjected to loads of different masses at different instances and to verify the robustness of this controller, the mass of link 1 is now altered to 1.2 kgs and tested with the tracking commands as mentioned in Eq. (6.1) and Eq. (6.2). The responses obtained are shown in Fig. 6.19 and the actuating commands in Fig. 6.20. The maximum value of actuating effort required is within the limits set for this system.

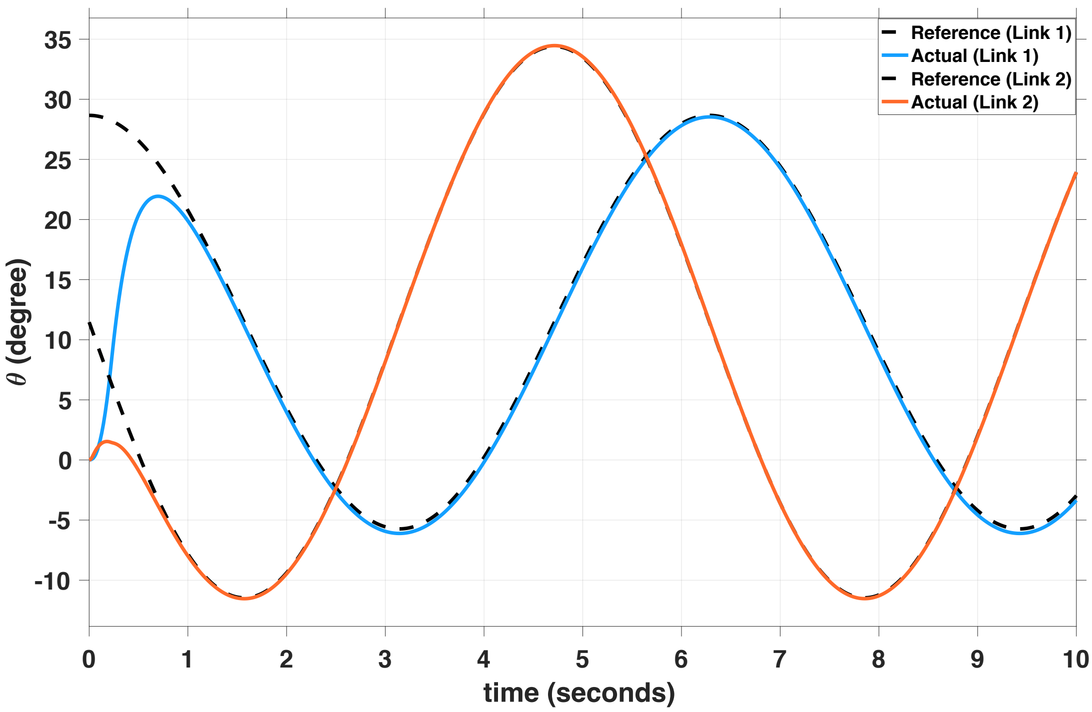


Fig. 6.19. Trajectory Tracking of θ_1 and θ_2 when mass of link 1 is subjected to variation

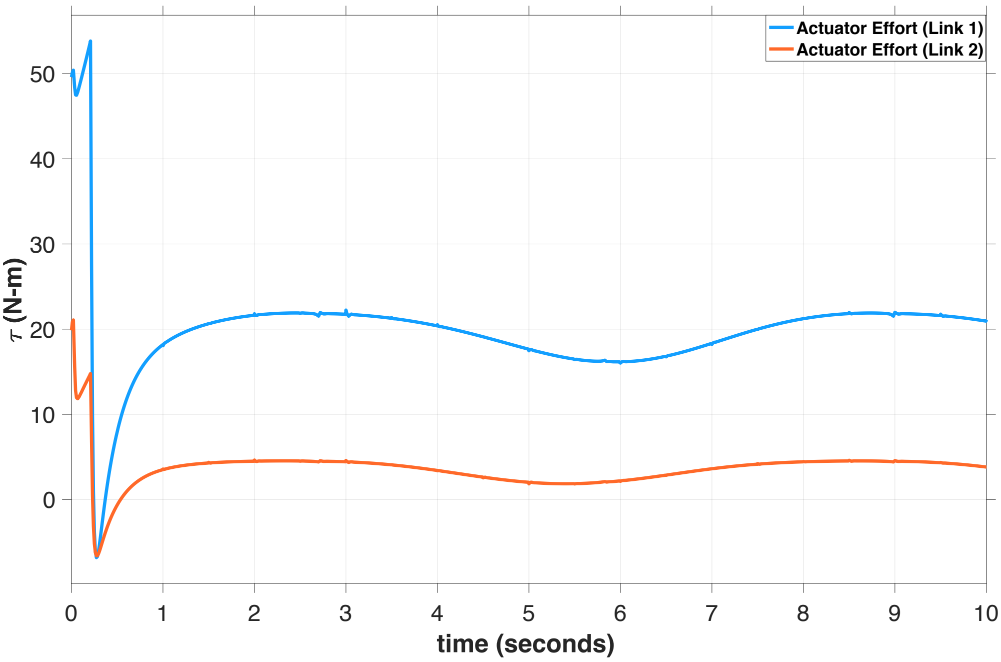


Fig. 6.20. τ_1 and τ_2 when mass of link 1 is subjected to variation

CHAPTER 7

CONCLUSION AND FUTURE WORK

7.1 CONCLUSION

This research presents a SMC controller for a 2-link robotic arm with saturation function to overcome chattering. The controller design quoted in Chapter 3 is simulated using MATLAB/Simulink environment. The controller is tested with trajectory tracking in terms of both slow varying and fast varying commands and the results show promising outcomes. The system is also subjected to a combination of disturbances and the disturbance handling capabilities of the controller is studied. The outcomes from the disturbance handling scenarios suggest that the controller is essentially robust. Also, the phenomenon of parametric uncertainties is tested for this controller and it can be confirmed that the controller is immune to certain parametric uncertainties and hence it can be concluded that the robust SMC controller is invariant of the system dynamics (point of consideration being nonlinearities and uncertainties) during the sliding phase. To sum up the proposed control structure has outstanding regulation with excellent steady state performance . It has acceptable response to fast varying commands. The control effort as a measure of energy expenditure is also minimal and most importantly the controller is essentially robust.

7.2 SCOPE FOR FUTURE WORK

- The controller has excellent trajectory tracking capabilities but needs improvement on regulation problems i.e the end effector positions vibrate when it is desired to hold at a particular angle. So a proper selection of controller gains or hybrid SMC control laws may be applied to overcome this.
- Various trajectories (different angular velocities) are handled better with specific controller gains. For example slow varying trajectories can work quite well with low values of controller gains hence minimising energy in the actuator efforts. So the methodology of Gain Scheduling can be implemented for optimally picking the gain sets.
- An empirical formulae can be obtained to get an idea about the gain sets based on plant parameters. Say a manipulator of mass m kg and length l can get a guideline to choose the controller gains depending upon its present parameters.
- Advanced methods like Feedback Linearization , Uncertainty and Disturbance Rejection techniques must be considered and implemented over the SMC controller
- Also, to ensure a full proof control the reaching phase can be ascertained with robustness .

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PUBLICATIONS

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SMC Tracking Control of 2-link Robotic Manipulator Subjected to External Disturbances

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***Abstract*— A robust sliding mode control (SMC) algorithm is designed which is used to control a 2-link robotic arm. The controller is tested for various types of disturbances and model parametric uncertainties. The novelty of the work lies in the fact that the designed controller is capable of handling slow varying disturbances, fast varying as well as unpredictable disturbances. Simulation results validate the accurate tracking capability and robust performance.**

Keywords—SMC control, Robust Controller, dynamic model, robotic manipulator, parametric uncertainty, disturbance rejection

I. Introduction

Control of robot manipulators is still a challenging control systems design problem today due to its high nonlinearity and strongly coupled robot dynamics [1]. The task gets even more complicated when the system is subjected to various unknown environments in the form of model uncertainties and unmeasurable external disturbances. With the use of the robots in critical applications like surgery, nuclear containments, industrial assembly lines etc., precise control of the robot arms has become an essential requirement. So, the figure of merit defining the controller is robustness and the modern-day design demands optimal control with minimal effort or energy to achieve a particular task. The end effectors of the robotic manipulators are to follow some desired trajectories as close as possible. Therefore, trajectory tracking problem is the most significant test to grade a controller design. A detailed survey of various strategies proposed in the literature for the design of robust controllers for robotic manipulators is mentioned in [2].

Ideally, the most common control systems design formulation is the arbitrary assignment of the system poles of a set of decoupled and linearized sub-system based on certain specifications. The essential requirements for such designs are that the system nonlinearities are neglected and the controller is acted upon with the most accurate dynamical model. Any mismatch due to parameter or modelling uncertainties will degrade the performance of the system and undermine the benefits of a controller. In most cases the uncertainty is assumed to be bounded by higher-order polynomials or approximated by some continuous functions [3]. Various model based robust control techniques have been presented in the literature for designing tracking controllers for robot manipulators. Some notable controllers include designs based on Proportional-derivative (PD) control [4], Proportional-integral-derivative [5], Lyapunov-based theory [6], optimal control [7], fuzzy logic controllers [8], neural networks [9] etc.

Sliding Mode Control (SMC) is a non-linear, robust control technique. The SMC based controller is resistant to uncertainties and parameter variations (if any), and does not demand for an exact model of the robotic arm, therefore a preferred choice for controlling the system of interest in this article. The SMC technique utilizes the theory of Variable Structure System (VSS) [10]. It involves driving an underlying state or the error dynamics of a state to be maintained at an attractive manifold, also called a sliding manifold due to which, the desired dynamic behaviour of the system can be asymptotically ascertained. The proceeding for a sliding mode control action involves two phases, the reaching phase and the sliding phase [11]. The reaching phase tries to align the required system state approach towards the sliding manifold while the sliding phase ensures the required dynamic behaviour is achieved [12]. During this sliding motion, the dynamics or properties of the system becomes invariant, thus making it less important to neutralise the system nonlinearities, which are otherwise required to be nullified if a conventional controller is used [13].The yielding dynamic motion of the states also becomes immune to certain parameter variations and external disturbances provided there are known bounds in those disturbances and variations [14]. However, the robustness property of the conventional sliding mode control with respect to variations of system parameters and external disturbances can only be achieved after the sliding phase is achieved. During the reaching phase, there is no guarantee for robustness [15]. An ideal control action from a sliding mode controller results in a discontinuous control action thus reflecting in a discontinuous actuator effort called chattering, which may demand high frequency switching actuators. The chattering phenomenon can be substantially mitigated by utilizing a linear saturation boundary layer function in the SMC algorithm [16]. One more advantage of SMC is that we can obtain sliding mode motion for higher order systems even if the controller is designed via reduced order modelling [17].

The organization of this article is as follows. In Section II, the mathematical formulations for the dynamics of a 2-link robotic arm are reviewed. Section III presents a sliding mode control algorithm and its integration with the system. Section IV illustrates the simulation results for the various test cases, thus justifying the designed controller. Finally, in section V, conclusions are presented.

II. Dynamic Modelling

This section formulates the mathematical equations for the statement of the problem which is the dynamical equations of motion of a 2-link robotic arm, allowed to swing

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