1   Introduction

Unmanned Aerial Vehicles (UAVs) are evolving as versatile machines in revolutionalising modern transportation systems. Their applications range from quick package delivery, seamless terrain mapping to emergency responsiveness and precision agriculture. However, UAVs carry fixed-capacity batteries, so an extended range of operation is always tricky. Also, UAVs are inefficient as nearly half of the battery power is consumed in supporting its own weight. One of the well-known problems concerning UAV operations is the path planning of a UAV with minimal energy consumption. A Power Consumption Model (PoCM) is used to estimate the endurance of a UAV, like its flying time, range, speed, payload limits, battery capacity, etc. So, an accurate PoCM plays a crucial role in planning the best possible path or trajectory in terms of energy being spent for a given operation.

In this research, we aim to address three aspects concerning the path planning of a quadrotor UAV. First, we want to derive a Power Consumption Model (PoCM) that can accurately predict the power consumed by a quadrotor in-flight. Second, using the obtained PoCM, we aim to perform path planning given an operation/objective such that the energy consumed in a trip is minimized. Third, we plan to devise an algorithm for path-planning in a multi-agent setup and dynamically changing environment such that a given objective is achieved with minimum power consumed by each agent.
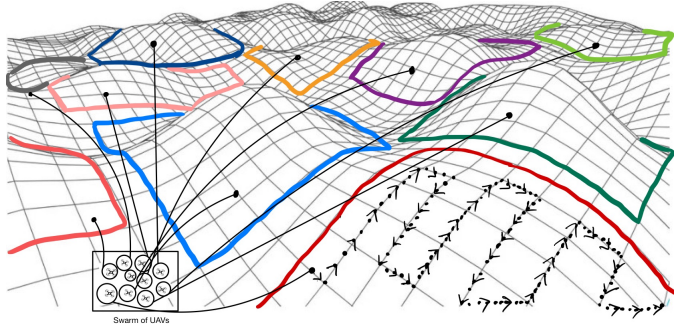

Figure 1: A quadrotor swarm surveying a 3D terrain

In this phase of work, we have formulated a power model for a quadrotor in a simulation environment. The model whose proceedings are enumerated in Section 2 reflects how the quadrotor consumes power for a predefined trajectory. The model is derived with a motivation that power consumed in a trip is dynamic and depends on the maneuvers a quadrotor undertakes in following a predefined trajectory. The fundamental maneuvers in a quadrotor flight are roll, pitch, yaw, ascend, descend and hover, and all other maneuvers are a combination of these maneuvers. We claim that tracking power based on maneuvers will lead to accurate power estimates. To prove this hypothesis, we have performed few practical field experiments which are highlighted in Section 3. Then, to step into path planning formulations, we have suggested a method for choosing trajectories that might lead to minimum energy consumption, which is described in Section 4. Finally, in Section 5, we have given the concluding remarks on how this progress will ultimately shape into the deliverables of this research.

2   A power model of a quadrotor based on maneuvers

To the best of our knowledge, almost all of the power models present in literature take in variables like velocities, body weight, payload weight, etc. and spit out power as a static number [1], but we claim that power drawn by the rotors varies with time and are not identical in all of the four rotors at all instants.
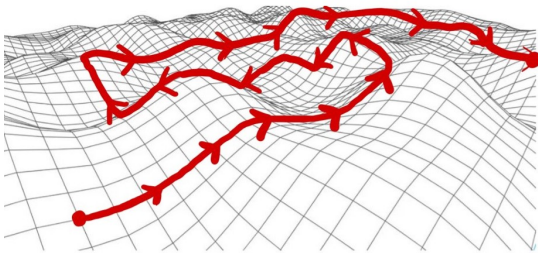

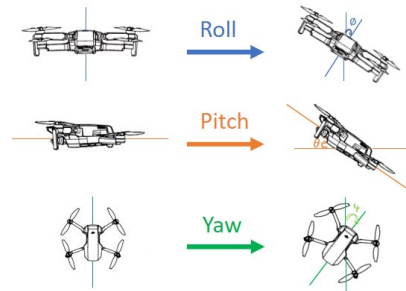Figure 2: A quadrotor travelling in a 3D terrain

Figure 3: Various maneuvers of a quadrotor

A quadrotor takes a sequence and combination of maneuvers (see Fig 3) to travel in a predefined trajectory (see Fig 2). Each maneuver is performed with a specific set of roll, pitch, and yaw angles, achieved with specific thrust and torque in each rotor. To produce a specific thrust, the rotors need to spin at a designated speed; hence, to execute these maneuvers, the rotor speeds are different in the four rotors. For example, to achieve a left roll maneuver, the right rotors spin faster than the left rotors. So, the power demand is different in all four rotors and hence power becomes a function of time and the trajectory it follows. Motivated by this idea, we have simulated a linearized dynamics of a quadrotor using MATLAB to see how each rotor power varies with time and derived a formulation to track the power in each rotor as the quadrotor moves in a trajectory. The following subsection enumerates our approach.

2.1   Power model design using MATLAB

Figure 4 summarises the dynamics governing a quadrotor's motion. We have incorporated a Proportional-Derivative (PD) control architecture to drive the error dynamics of the quadrotor motion. In this formulation, the model takes the quadrotor parameters (mass, arm length, inertia tensor, gravity), predefined trajectory (in the form of a twice differentiable polynomial) and the controller gains as input and outputs the individual rotor powers as a function of time. We achieve it by probing into the controller inputs $u_1(t), u_2(t)$ in order to calculate instantaneous thrust $F_k(t)_{k \in \{1,2,3,4\}}$. Finding the instantaneous thrusts help us in realizing the maneuvers that the quadrotor undertakes. Considering the ideal case where thrust $F \propto \omega^2$, we obtain the corresponding rotor speeds $\omega_k(t)_{k \in \{1,2,3,4\}}$. The goal

is to obtain a rotor's current and voltage profile from a given rotor speed data to calculate the power profiles. So, we have evaluated an inverse map from the governing second-order equations of a DC motor (where we consider rotor parameters as R_a, L_a, B, J, K_m) which is used to calculate the current profiles $i_k(t)_{k\in\{1,2,3,4\}}$ from the given rotor speed profiles $\omega_k(t)_{k\in\{1,2,3,4\}}$. Considering voltage supplied to the motors as $v(t)$, we then obtain the instantaneous power profiles as $P_k(t)_{k\in\{1,2,3,4\}} = v.i_K(t)$. An example showing the power calculation of a quadrotor flight using our formulation is discussed in the next subsection. In this case, we have assumed nominal values of the quadrotor attributes and motors parameters.

Quadrotor Dynamics

The translational motion is given by Newton's Laws

$$m\begin{bmatrix}\ddot{x}\\\ddot{y}\\\ddot{z}\end{bmatrix} = \begin{bmatrix}0\\0\\-mg\end{bmatrix} + R\begin{bmatrix}0\\0\\\underbrace{F_1+F_2+F_3+F_4}_{u_1}\end{bmatrix}$$

The rotational motion is given by Euler equations

$$I\begin{bmatrix}\dot{p}\\\dot{q}\\\dot{r}\end{bmatrix} = \underbrace{\begin{bmatrix}0 & L & 0 & -L\\-L & 0 & L & 0\\\gamma & -\gamma & \gamma & -\gamma\end{bmatrix}\begin{bmatrix}F_1\\F_2\\F_3\\F_4\end{bmatrix}}_{u_2} - \begin{bmatrix}p\\q\\r\end{bmatrix}\times I\begin{bmatrix}p\\q\\r\end{bmatrix}$$

The control laws are given by

$$\boldsymbol{u_2} = \begin{bmatrix}k_{p,\phi}\left(\phi_{des}-\phi\right)+k_{d,\phi}\left(p_{des}-p\right)\\k_{p,\theta}\left(\theta_{des}-\theta\right)+k_{d,\theta}\left(q_{des}-q\right)\\k_{p,\phi}\left(\psi_{des}-\psi\right)+k_{d,\psi}\left(r_{des}-r\right)\end{bmatrix} \quad \begin{aligned}\phi_{\mathrm{des}} &= \tfrac{1}{g}\left(\ddot{r}_{1,\,\mathrm{des}}\sin\psi_T - \ddot{r}_{2,\,\mathrm{des}}\cos\psi_T\right)\\\theta_{\mathrm{des}} &= \tfrac{1}{g}\left(\ddot{r}_{1,\,\mathrm{des}}\cos\psi_T + \ddot{r}_{2,\,\mathrm{des}}\sin\psi_T\right)\end{aligned}$$

$$\left(\ddot{r}_{i,des}-\ddot{r}_{i,T}\right)+k_{d,i}\left(\dot{r}_{i,des}-\dot{r}_i\right)+k_{p,i}\left(r_{i,des}-r_i\right)=0$$

$$u_1 = mg + m\ddot{r}_{3,des} = mg - m\left(k_{d,3}\dot{r}_3 + k_{p,3}\left(r_3-r_{3,0}\right)\right)$$

The inertial frame **A** is defined by axes $\{a_1,a_2,a_3\}$ and the body frame **B** is defined by axes $\{b_1,b_2,b_3\}$. **O** is the origin.

$\{a_1,a_2,a_3\}$ coincides with world $\{x,y,z\}$

$\{b_1,b_2,b_3\}$ are also the principal axes

The length of each quadrotor arm is **L** m

r=[r_1,r_2,r_3]' = [x,y,z]' is the position vector

$\{\Phi,\Theta,\psi\}$ are the Roll, Pitch and Yaw angles

p,q,r are the components of angular velocity in the body frame

$\mathcal{A}_{\omega_B} = p\mathbf{b}_1 + q\mathbf{b}_2 + r\mathbf{b}_3$

$$\begin{bmatrix}p\\q\\r\end{bmatrix}=\begin{bmatrix}c\theta & 0 & -c\phi s\theta\\0 & 1 & s\phi\\s\theta & 0 & c\phi c\theta\end{bmatrix}\begin{bmatrix}\dot{\phi}\\\dot{\theta}\\\dot{\psi}\end{bmatrix}$$

$F_i = k_F\omega_i^2 \qquad \gamma = \frac{k_M}{k_F}$

$M_i = k_M\omega_i^2$

m = mass of the quadrotor

R = Rotation matrix from B to A

g = acceleration due to gravity

K_F = thrust constant

K_M = torque constant

$F_{i,i=\{1,2,3,4\}}$ = Thrust in rotor i

$\omega_{i,i=\{1,2,3,4\}}$ = Speed of rotor i

I = Inertia tensor

$\{k_{p,\phi},k_{d,\phi}\},\{k_{p,\theta},k_{d,\theta}\},\{k_{p,\psi},k_{d,\psi}\}$ = Attitude control gains

$\{\phi_{des},\theta_{des},\psi_{des}\}$ = Desired roll, pitch, yaw

$\{r_{i,des}^{\cdot\cdot},r_{i,des}^{\cdot},r_{i,des}\}_{,i=\{x,y,z\}}$ = Desired translational acceleration, velocity, position

$\{k_{p,i},k_{d,i}\}_{,i=\{x,y,z\}}$ = Position control gains

$$\boldsymbol{S} = \begin{bmatrix}x\\y\\z\\\dot{x}\\\dot{y}\\\dot{z}\\q_w\\q_x\\q_y\\q_z\\p\\q\end{bmatrix}$$
state vector

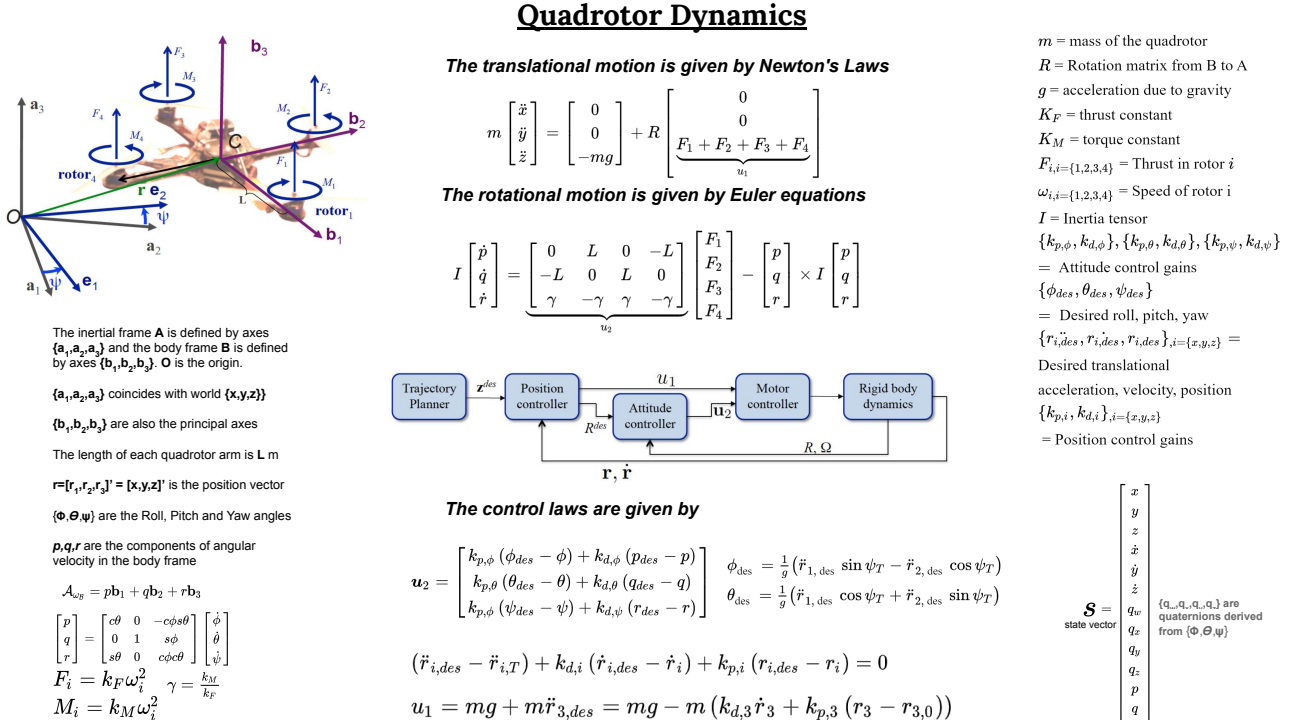$\{q_w,q_x,q_y,q_z\}$ are quaternions derived from $\{\Phi,\Theta,\psi\}$

Figure 4: Dynamics of a quadrotor motion

2.2 Power calculations using the designed model

Here, we discuss three quadrotor flight simulations to reflect pitch only, roll only and hover maneuver in order to highlight the power implications in each case. We consider $\ddot{r}_{des} = 0.375t^3 - 0.00285t^4 + 0.0001t^5$, $\dot{r}_{des} = 0.1125t^2 - 0.0112t^3 + 0.0005t^4$, $r_{des} = 0.225t - 0.0336t^2 + 0.002t^3$. To drive the quadrotor with only pitch and only roll maneuver, we use the commanded trajectory as $\{\ddot{r}_{x,des} = \ddot{r}_{des}, \dot{r}_{x,des} = \dot{r}_{des}, r_{x,des} = r_{des}, \ddot{r}_{i\in\{y,z\},des} = \dot{r}_{i\in\{y,z\},des} = r_{i\in\{y,z\},des} = 0\}$ and $\{\ddot{r}_{y,des} = \ddot{r}_{des}, \dot{r}_{y,des} = \dot{r}_{des}, r_{y,des} = r_{des}, \ddot{r}_{i\in\{x,z\},des} = \dot{r}_{i\in\{x,z\},des} = r_{i\in\{x,z\},des} = 0\}$ respectively. Similarly for hover, we use the commanded trajectory as $\{\ddot{r}_{i\in\{x,y,z\},des} = \dot{r}_{i\in\{x,y,z\},des} = r_{i\in\{x,y\},des} = 0, r_{z,des} = 11\}$. Each of the flight simulations is performed for 10 seconds. The physical parameters of the quadrotor are $\{m = 0.18kg, L = 0.018m, I = \begin{bmatrix}0.0025 & 0 & 0\\0 & 0.000232 & 0\\0 & 0 & 0.0003738\end{bmatrix}$ kg.m.m$\}$, the controller gains are as: Attitude Control: $\{k_{p,\phi}, k_{p,\theta}, k_{p,\psi}, k_{d,\phi}, k_{d,\theta}, k_{d,\psi}\} = \{100, 100, 100, 2, 2, 2\}$, Position Control: $\{k_{p,x}, k_{p,y}, k_{p,z}, k_{d,x}, k_{d,y}, k_{d,z}\} = \{200, 200, 100, 40, 40, 20\}$ We consider a DC motor having motor parameters as $\{R_a = 1\Omega, L_a = 1H, J = 5Kg.m.m, B = 0.01N.m.s, K = 1.6V/rad/s, K_F = K_M = 1\}$ and the voltage supplied to the motor as $1V$. For each simulated flight, we get thrust $F_{i\in\{1,2,3,4\}}$, rotor speeds $\omega_{i\in\{1,2,3,4\}}$ and power data $P_{i\in\{1,2,3,4\}}$ as a function of time. To summarize these data, we have calculated the RMS value of each time series data, which has been tabulated in Table I.

Maneuver	Thrust (N)				Rotor Speed (RPM)				Rotor Power(W)				Total Power (W)	Difference(%)
	F1	F2	F3	F4	ω1	ω2	ω3	ω4	P1	P2	P3	P4		
Hover	0.4405	0.4405	0.4405	0.4405	0.6594	0.6594	0.6594	0.6594	0.6551	0.6551	0.6551	0.6551	2.6204	-
Pitch	0.5177	0.5181	0.5571	0.5181	0.6994	0.7151	0.7304	0.7151	0.6846	0.7117	0.7203	0.7117	2.8283	8
Roll	0.5063	0.5720	0.5063	0.5442	0.7048	0.7150	0.7048	0.6984	0.7019	0.6977	0.7019	0.6761	2.7776	6

It is seen that power consumed during pitch and roll is 8% and 6% higher than that during hover. A traditional power model could not have differentiated roll and pitch in terms of power, as the horizontal velocity is the same in both cases. This is also observed from the power data obtained from field experiments (shown in the next section). Based on this formulation, we want to outline that in order to estimate the power that the quadrotor will consume when following a predefined trajectory, we must look into the complete power profiles of four rotors. Such a method might lead to accurate power tracking and produce efficient trajectories through path planning.

3 Field Experiments to identify power basaed on maneuvers

We have performed a few field experiments using a DJI Air 2 drone to shed light on our hypothesis of looking into power based on manoeuvres. In each experiment, the drone is made to perform a specific maneuver for a period of 16 seconds. The power data is extracted from the log files populated on the motherboard of the drone. For these experiments and average power consumed in each maneuver are summarized in Table I. In most maneuvers, we observe significant power differences in the four rotors which supports our approach of looking into power basesd on maneuvers. We have also compared the power consumed in these maneuvers with the power estimates from the model derived by [2] (see Table II). We observe that the power model in [2] was not able to accurately capture the power estimations in most cases.

TABLE II: Avg power for various maneuvers using DJI Air 2

Maneuver	Left Front	Right Front	Left Back	Right Back	Total Power(W)
		Motor Power (W)			
Hover	38.01	42.16	33.37	31.8	145.34
Pitch(F)	38.09	42.26	52.32	53.53	186.2
Pitch(B)	49.31	52.76	24.76	28.05	154.88
Roll(L)	34.26	57.04	36.03	43.67	171
Roll(R)	58.12	38.22	43.67	41.72	181.73
Ascend	57.24	50.36	42.2	40.7	190.5
Descend	42.7	35.3	31.7	27.09	136.79
Ascend+Pitch(B)	69.59	70.21	39.33	39	218.13
Descend+Pitch(F)	27.87	30.73	31.06	31.84	121.5
Yaw(CL)	40.14	45.81	34.66	34.58	155.19

TABLE III: Comparison with a power model

Maneuver	(Horizontal velocity, Vertical Velocity) (m/s, m/s)	Total Power (W)	Power from [2] (W)	Power Difference (W)	% Power difference
Hover	(0,0)	145.34	145.35	-0.01	-0.00
Pitch(F)	(11.9,0)	186.2	186.86	-0.66	-0.35
Pitch(B)	(11.98,0)	154.88	187.86	-32.98	-21.29
Roll(L)	(12.01,0)	171	188.25	-17.25	-10.08
Roll(R)	(11.1,0)	181.73	177.77	3.96	2.18
Ascend	(0,4.14)	190.5	186.18	4.32	2.27
Descend	(0,-3.02)	136.79	131.87	4.92	3.59
Ascend + Pitch(B)	(8.61,3.99)	218.13	203.33	14.8	6.78
Descend + Pitch(F)	(11.1,-4.9)	121.5	152.87	-31.37	-25.81
Yaw(CCL)	(0,0)	156.15	145.35	10.8	6.91
Yaw(CL)	(0,0)	155.19	145.35	9.84	6.34
Pitch(F)	(16.38,0)	297.44	273.52	23.92	8.04
Pitch(B)	(16.82,0)	230.9	285.78	-54.88	-23.76
Roll(R)	(18.4,0)	262.96	336.06	-73.1	-27.79
Roll(L)	(18.94,0)	244.67	355.58	-110.91	-45.33
Ascend + Pitch(B)	(15.14,4.02)	275.64	310.55	-34.91	-12.66
Descend + Pitch(F)	(17.95,-5)	187.37	270.18	-82.81	-44.19

4 Proceedings for Path Planning

Our goal is to minimize the energy that will be consumed in a trip. We achieve it by evaluating how expensive a trajectory is in terms of power using the power model derived in Section 2. In this phase of work, we have evaluated minimum jerk trajectories i.e the trajectories obtained by minimizing $\dddot{r}^2$, where r is the position of the quadrotor at any instant. To this end, we have described a case where we have commanded the quadrotor to reach $11m$ along the $x-axis$ with two separate trajectories (shown in Figure 5) in which Trajectory 2 is formulated with a minimum jerk action. The power estimates are calculated using our model which is summarized in Table IV. We see that Trajectory 2 consumes 9.4% less power compared to Trajectory 1. (Note : Model parameters are same as that of Section 2.2)
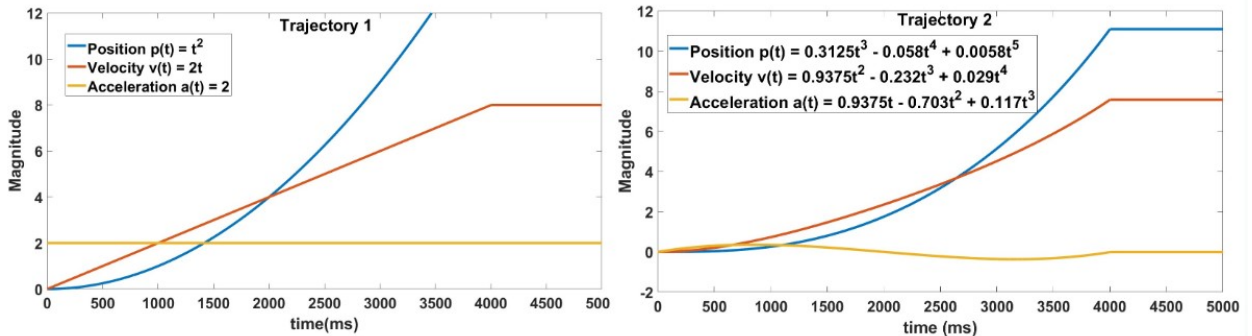

Figure 5: Trajectory Plots

TABLE IV: Power attributes of the trajectories

Trajectory	Time of convergence (s)	P1	P2	P3	P4	Total Power (W)
		Rotor Power (W)				
Trajectory 1	3.3	0.6695	0.7126	0.6889	0.7126	2.7836
Trajectory 2	4	0.4796	0.6602	0.7493	0.6602	2.5439

5 Conclusion

- The idea of deriving power based of maneuvers might be accurate because it helps us estimate each of the instantaneous rotor power and as seen in the simulation model and field experiments, there are significant differences among individual instantaneous rotor powers. In the next phase of our work, we aim at deriving the power model based on the non-linear dynamics of the quadrotor. We are also working on measuring the inertia tensor of the DJI Air 2 drone and obtaining the current-speed relationship of its propellers so that we can apply our model in estimating its power.

- We will also step into applying a more realistic path-planning formulation by modelling practical constraints and environmental factors and then extend our work into a multi-agent setup.

References

[1] J. Zhang, J. F. Campbell, and A. C. Sweeney II, "Energy consumption models for delivery drones: A comparison and assessment," *Transportation Research Part D: Transport and Environment*, vol. 90, p. 102668, 2021.

[2] Z. Liu, R. Sengupta, and A. Kurzhanskiy, "A power consumption model for multi-rotor small unmanned aircraft systems," in *2017 International Conference on Unmanned Aircraft Systems (ICUAS)*. IEEE, 2017, pp. 310–315.