Annual Progress Seminar

Path Planning of an UAV with Minimal Energy Consumption

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Introduction

Use Cases of UAVs

Parcel Delivery

Precision Agriculture

Land Survey

Mapping

Photography

Weather Forecast

UAVs in India

- UAVs have immense scope in the field of agriculture and border patrolling. *[Online article FICCI]*
- The Civil Aviation Ministry estimates India's drone sector to become a ₹ 120 -150 billion industry by 2026. *[Online article IBEF]*
- “The Drone Rules 2021” has liberalised the drone rules and aims to encourage more applications using UAVs.

Technical Drawbacks with UAVs

- Inefficient mode of transport
- Limited flying time
- Power Consumption estimation is exhaustive and inaccurate
- Path Planning is complicated in implementation

Problem Statement

Path Planning of a UAV with minimum energy consumption

- **Power Consumption Model**

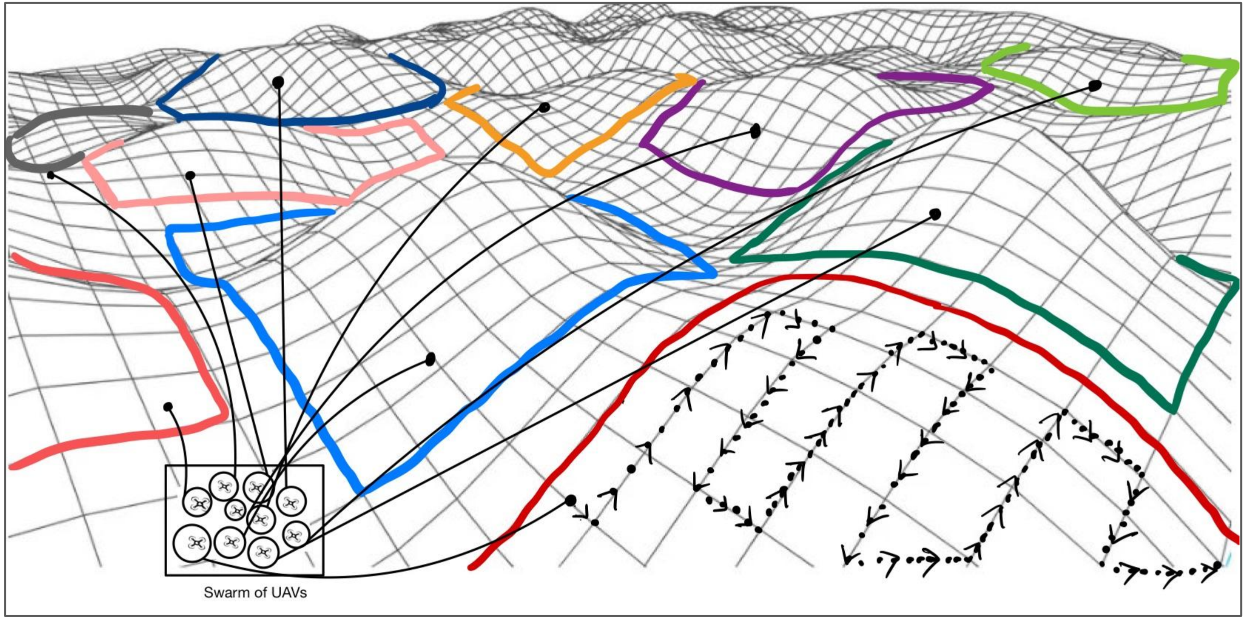
  Flying time
  Range
  Speed
  Payload

- **Path Planning**

  Routes
  Number of UAVs
  Battery Capacity

- **Multi-agent algorithms**

  Environmental Dynamics
  External Constraints



A quadrotor swarm surveying a 3D terrain

Power Consumption Modelling

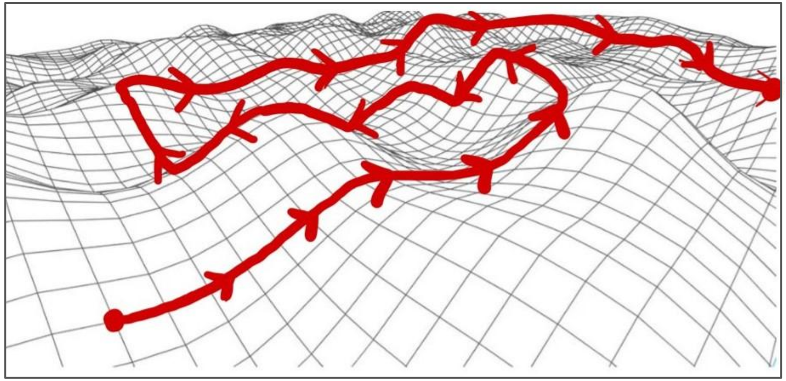
Parameters affecting power of a UAV [1],[2],[3]

- **UAV Design**
  - UAV weight
  - Number of rotors
  - Number of blades per rotor
  - Total propeller area
  - Blade chord length
  - Angle of attack of propeller disk
  - Advance ratio of propellers
  - Size of UAV
  - UAV body drag coefficients
  - Battery weight
  - Battery Capacity
  - Size of battery
  - Power transfer efficiency
  - Maximum speed
  - Maximum payload
  - Lift-to-drag ratio
- **Environment**
  - Air density
  - Gravity
  - Wind velocity
  - Wind incident angle
  - Weather
  - Ambient temperature
- **Drone dynamics**
  - Airspeed (vertical and horizontal)
  - Motion (take-off, landing, hover, levelled flight)
  - Acceleration/Deceleration
  - Roll/Pitch/Yaw angle
  - Rotor speeds
  - Flight angle
  - Flight altitude
- **Delivery Operations**
  - Payload weight
  - Size of payload
  - Drag coefficient of payload
  - Fleet size and mix
  - Single/multi stop trip
  - Delivery mode (tether, landing, parachute)
  - Area of service region

[1] J. Zhang, J. F. Campbell, D. C. Sweeney II, and A. C. Hupman, Energy consumption models for delivery drones: A comparison and assessment," *Transportation Research Part D: Transport and Environment,* vol. 90, p. 102668, 2021.
[2] Z. Liu, R. Sengupta, and A. Kurzhanskiy, \A power consumption model for multirotor small unmanned aircraft systems," in *2017 International Conference on Un- manned Aircraft Systems (ICUAS)*. IEEE, 2017, pp. 310{315.
[3] K. Dorling, J. Heinrichs, G. G. Messier, and S. Magierowski, \Vehicle routing problems for drone delivery," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* vol. 47, no. 1, pp. 70{85, 2016.

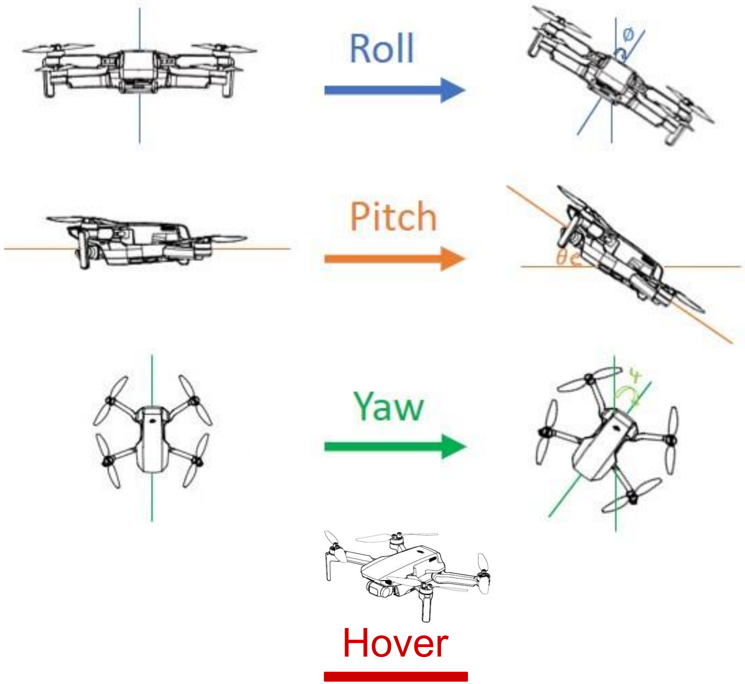
Existing Power Models

- Power is given as a static number [1]
- Oversimplification of deriving Power from Hovering [2]
- Power formulation on straight line trajectories
- Encapsulating aerodynamics aspects with constants [3]
- Lack of consensus in predicting power with different power models [4]



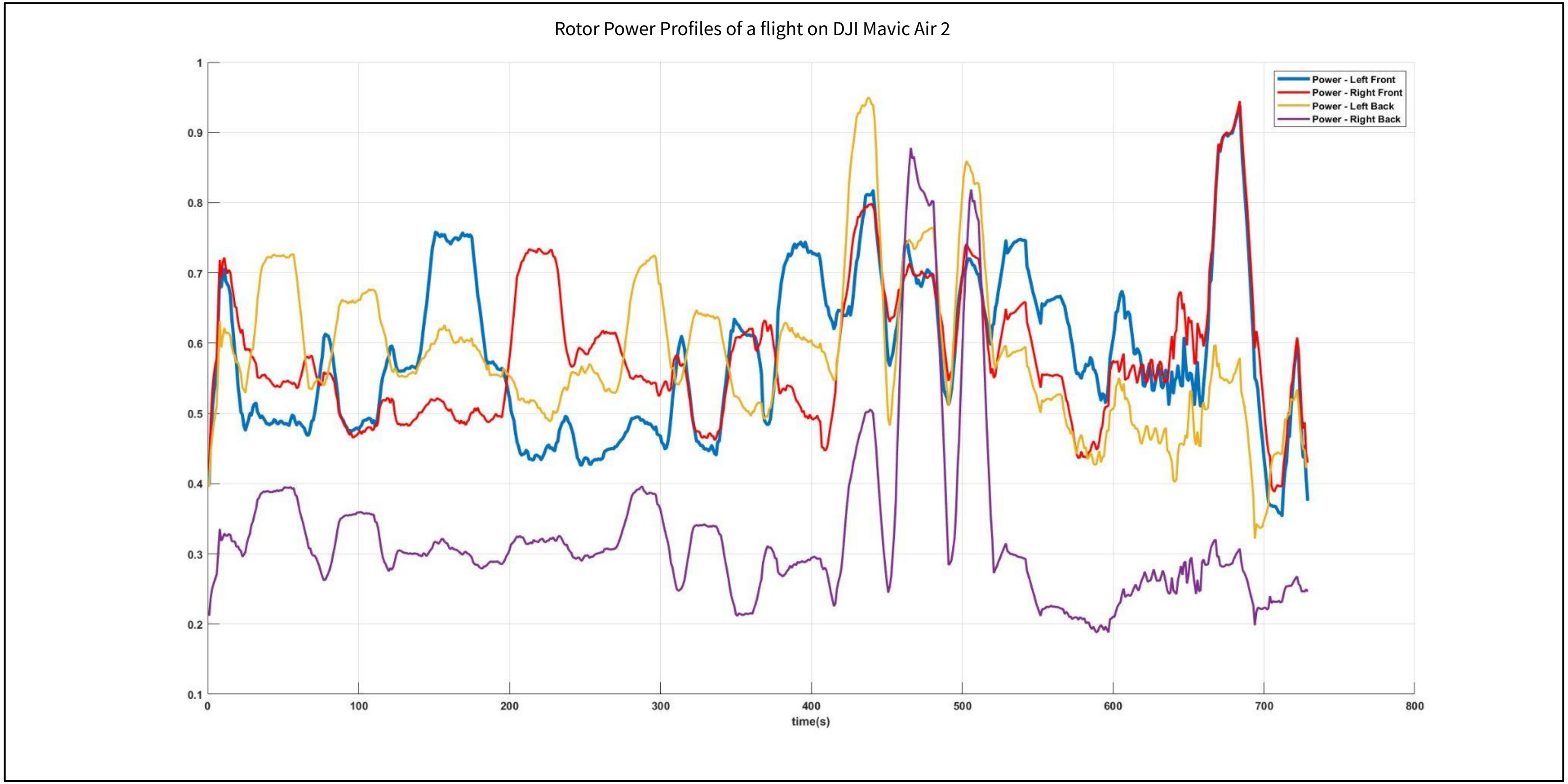
A quadrotor travelling in a 3D terrain

Maneuvers



[1] Liu et. al., "A power consumption model for multirotor small unmanned aircraft systems," in *2017 ICUAS*. IEEE, pp. 310–315. **[2]** R. D'Andrea, "Guest editorial can drones deliver?" *IEEE Transactions on Automation Science and Engineering*, pp. 647–648, **[3]** J. Leishman, Principles of Helicopter Aerodynamics: 12 (Cambridge Aerospace Series, Series Number 12). Cambridge, UK: Cambridge University Press, 2002. **[4]** J. Zhang, J. F. Campbell, D. C. Sweeney II, and A. C. Hupman, *"Energy consumption models for delivery drones: A comparison and assessment," Transportation Research Part D: Transport and Environment,* vol. 90, p. 102668, 2021.

Power Consumption Modelling
Motivation



Power Consumption Modelling

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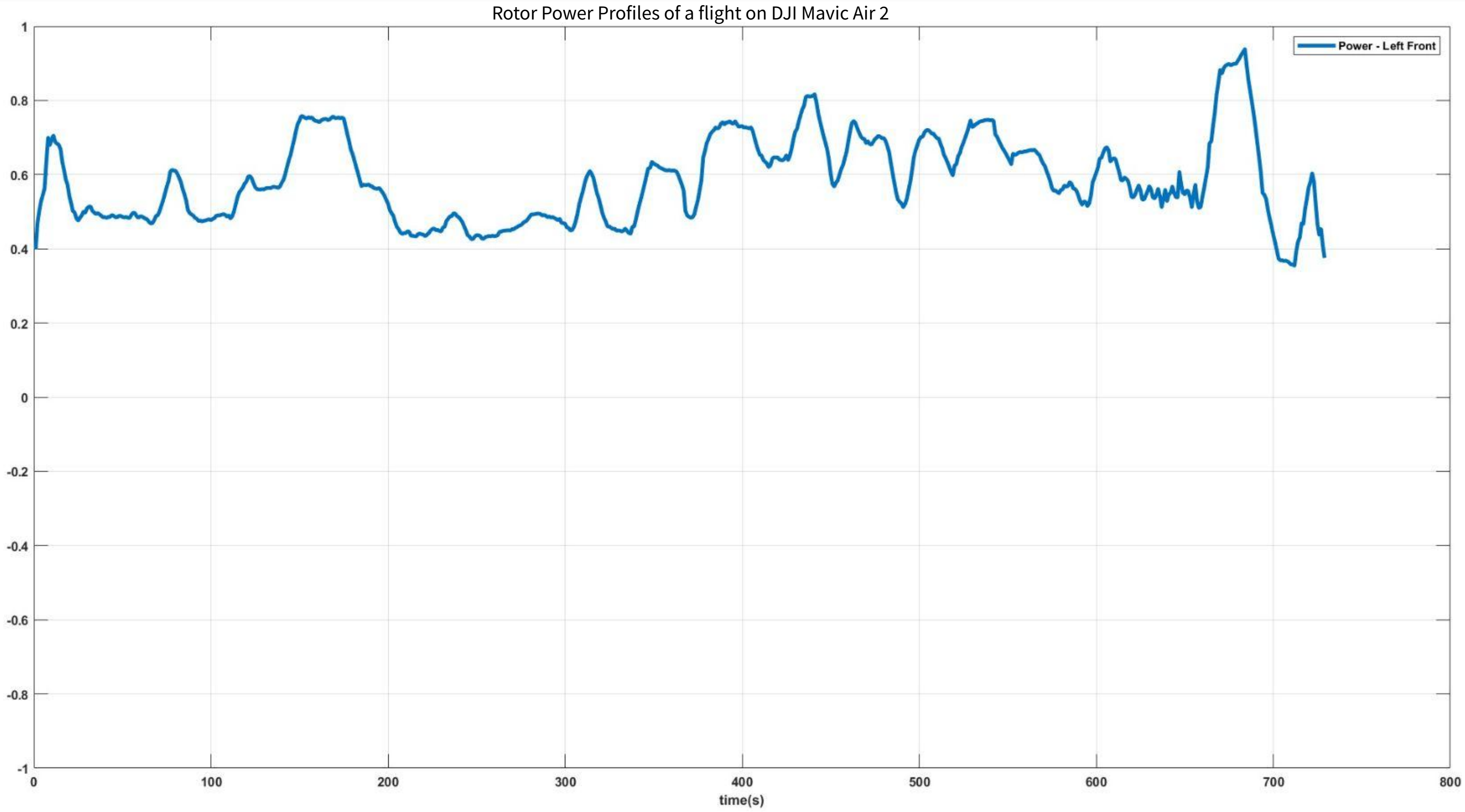

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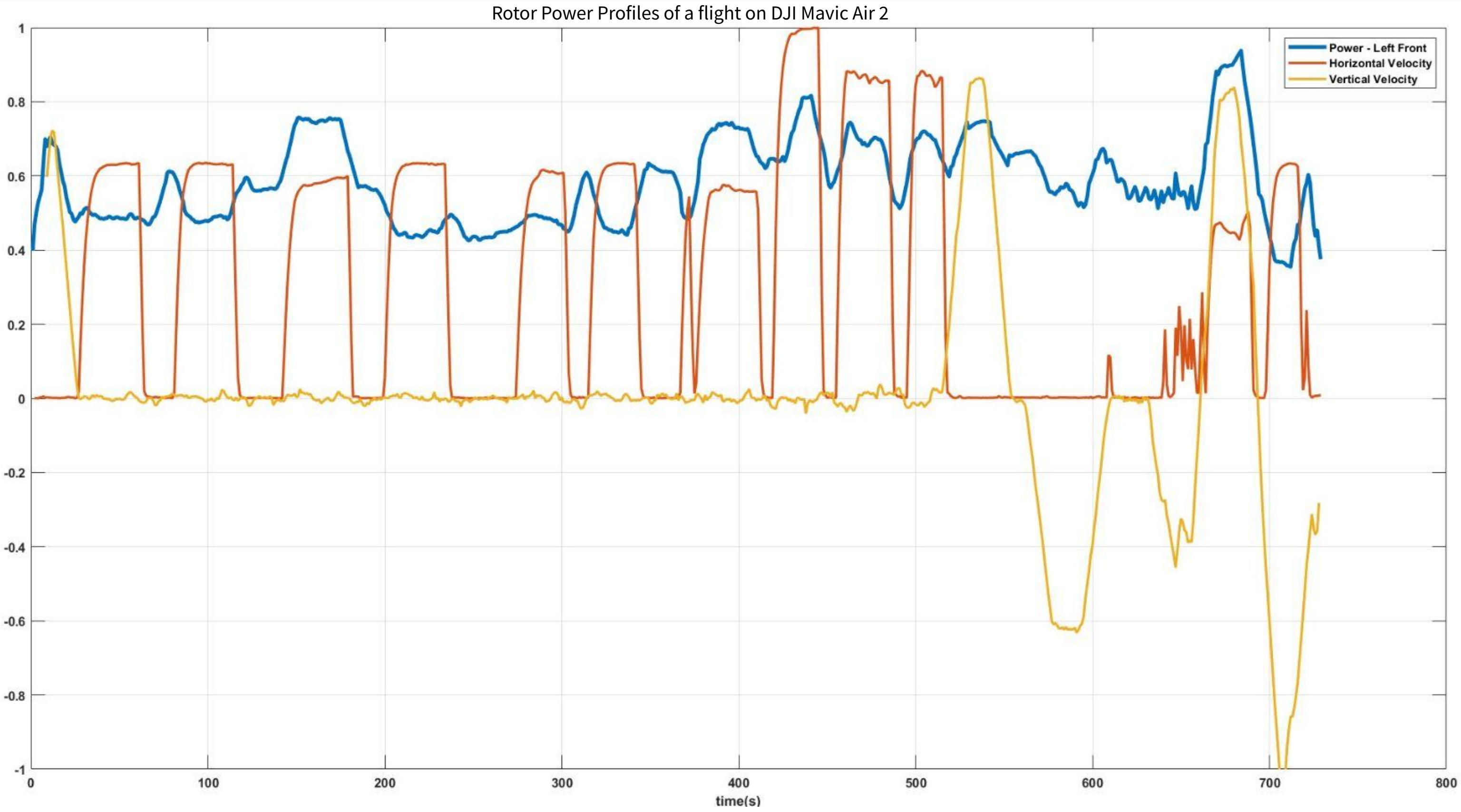

Power Consumption Modelling

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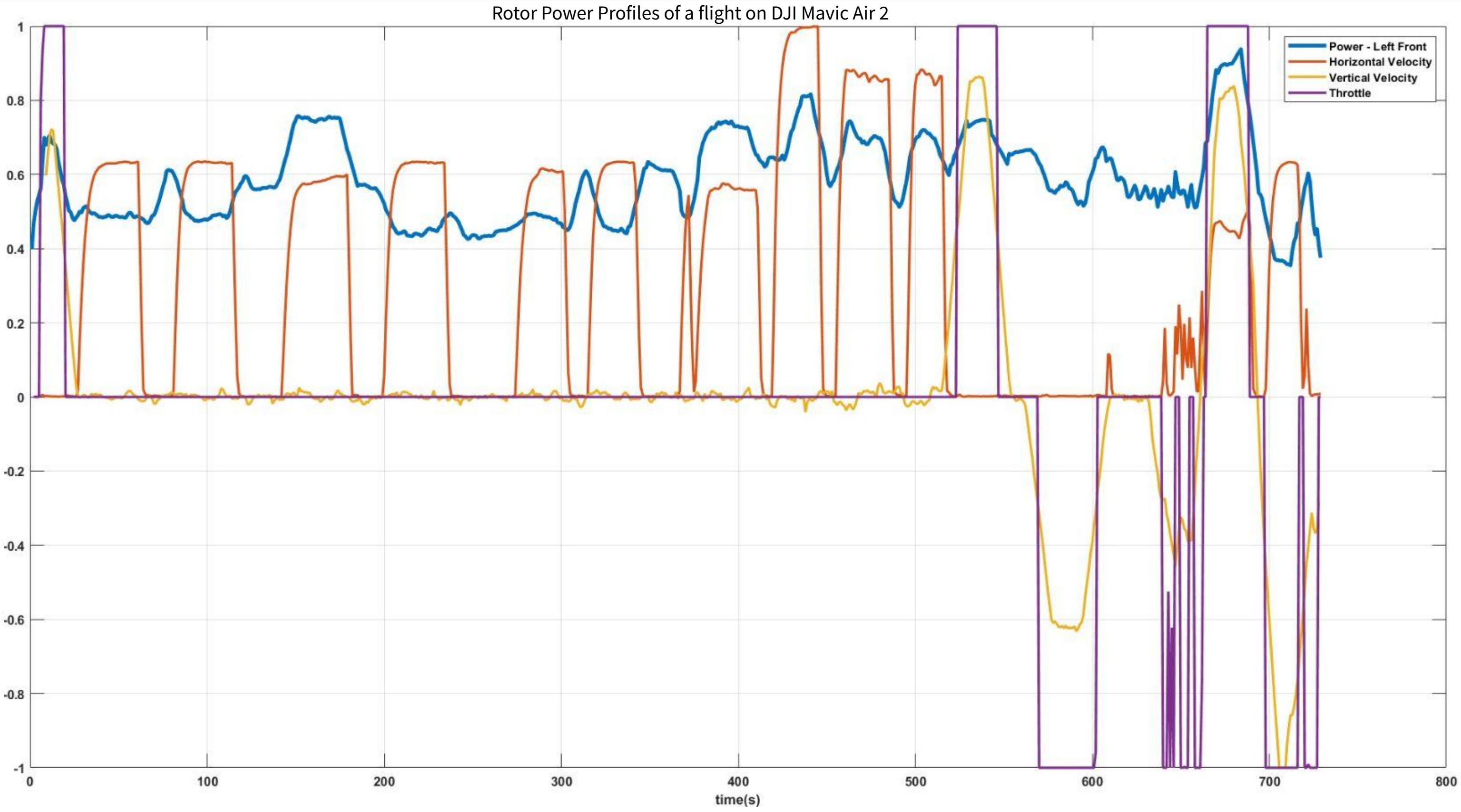
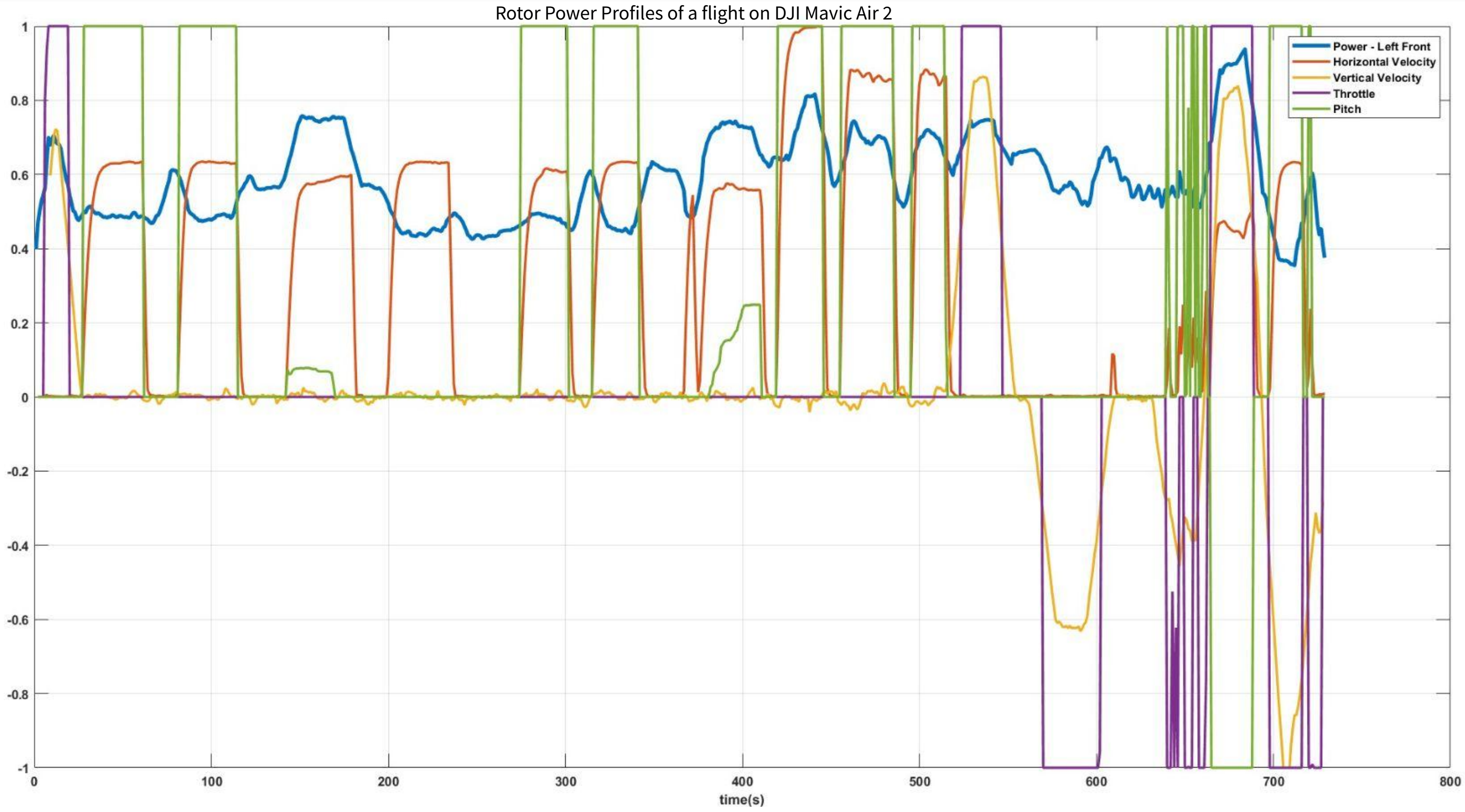


Power Consumption Modelling

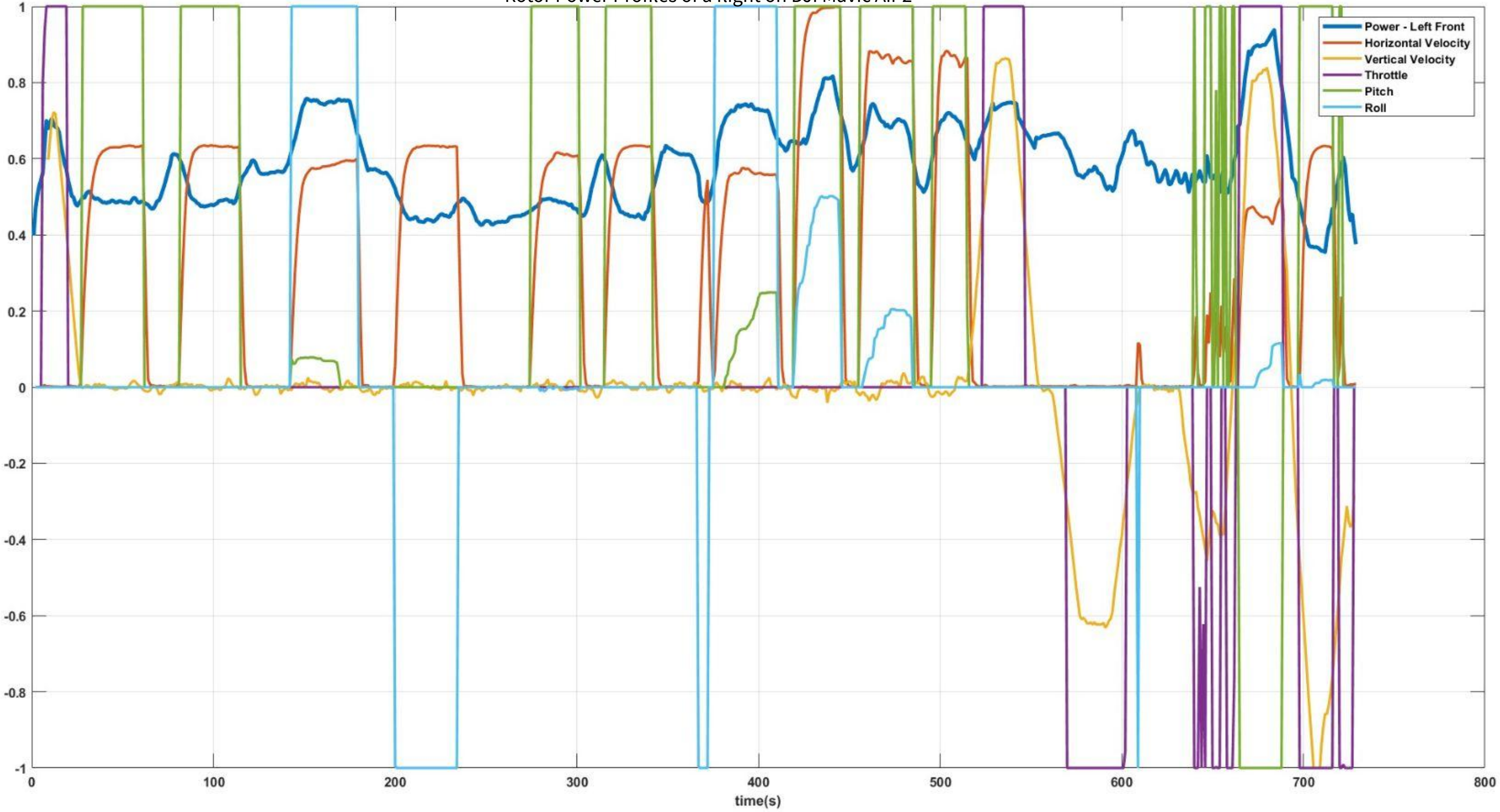
Motivation



Power Consumption Modelling

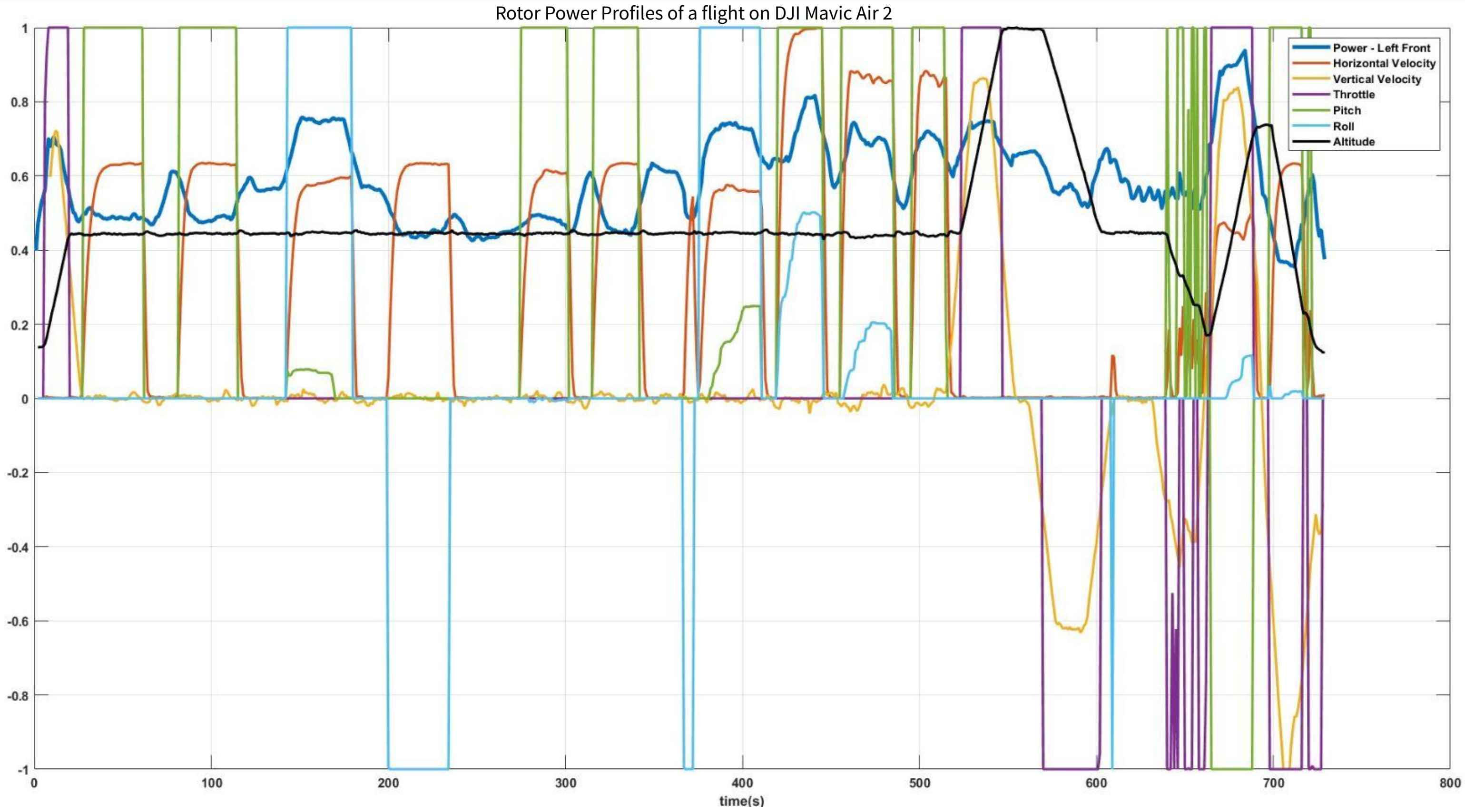
Motivation

Rotor Power Profiles of a flight on DJI Mavic Air 2

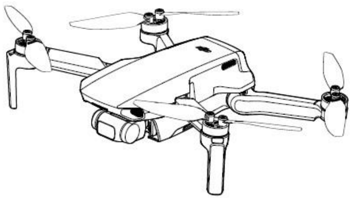


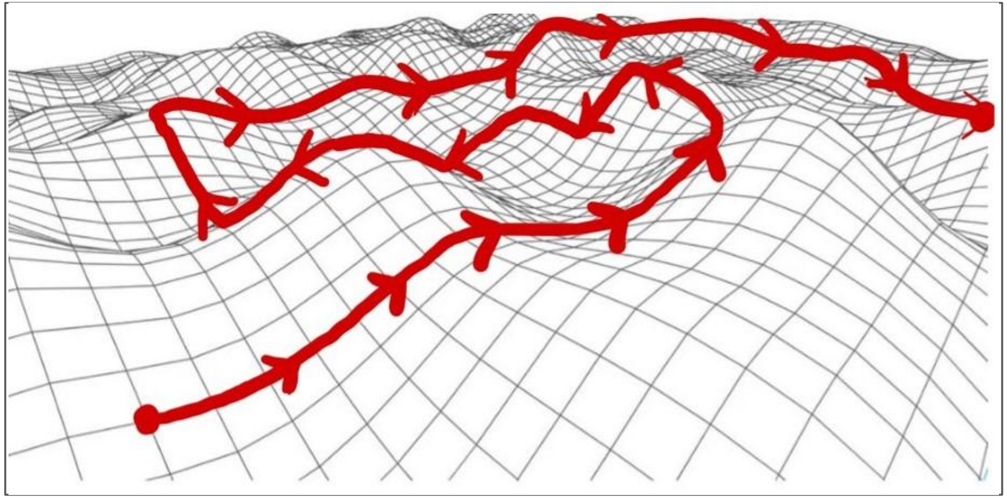
Power Consumption Modelling

Motivation

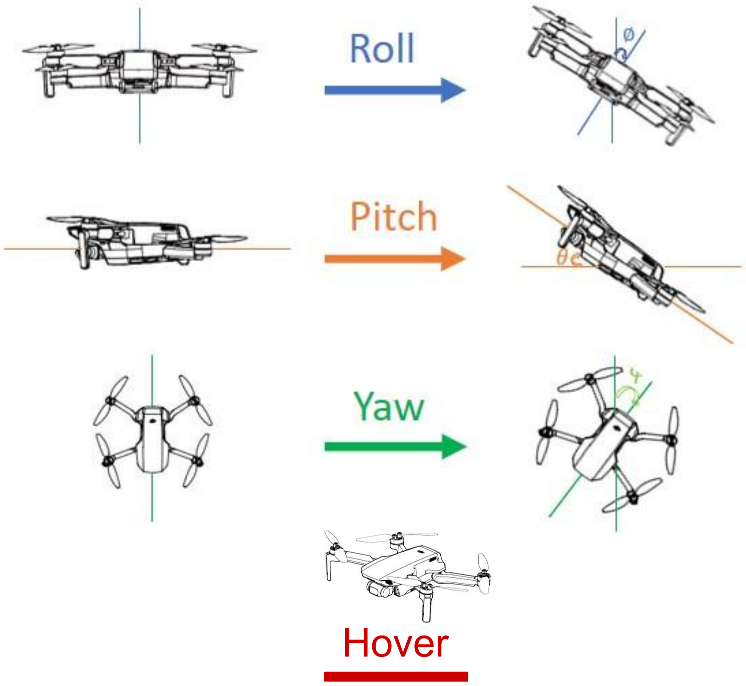


Power Consumption Model for a quadrotor



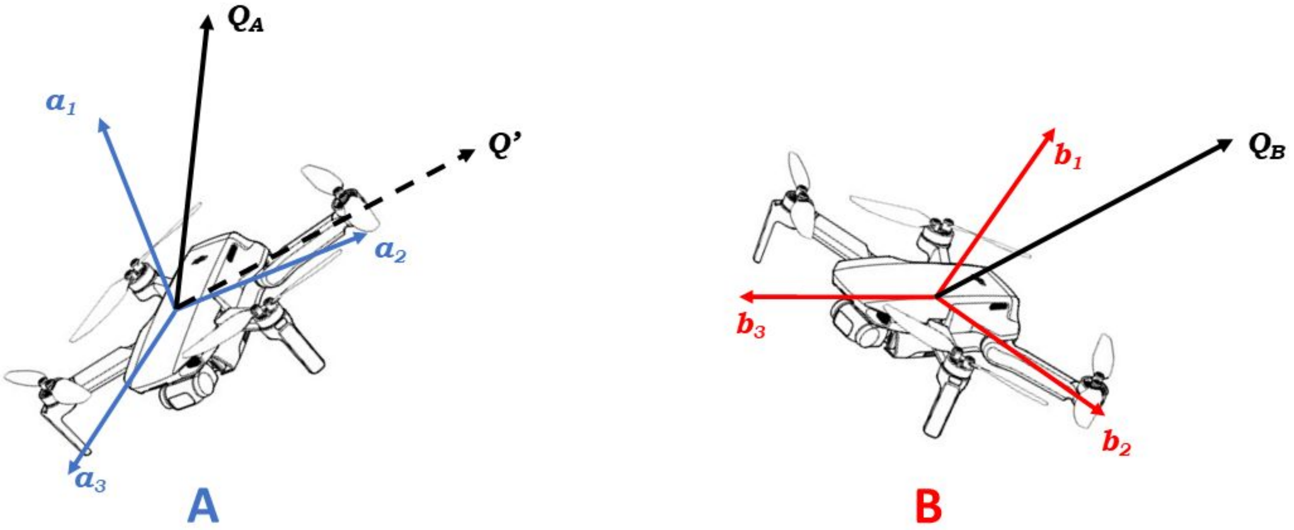


Maneuvers

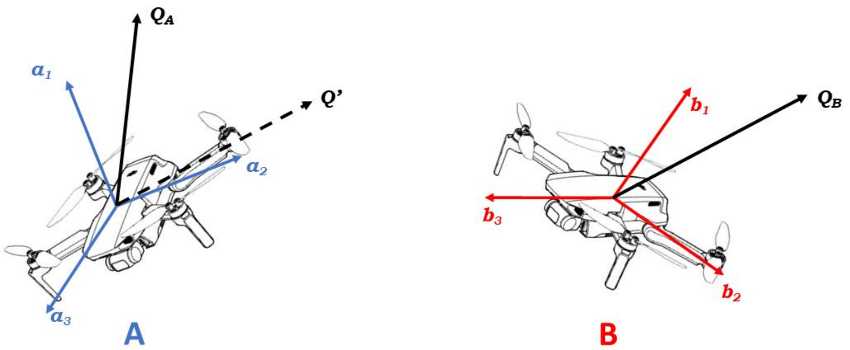


Power Model of a quadrotor
Preliminaries

Reference Frames



Rotation Matrix

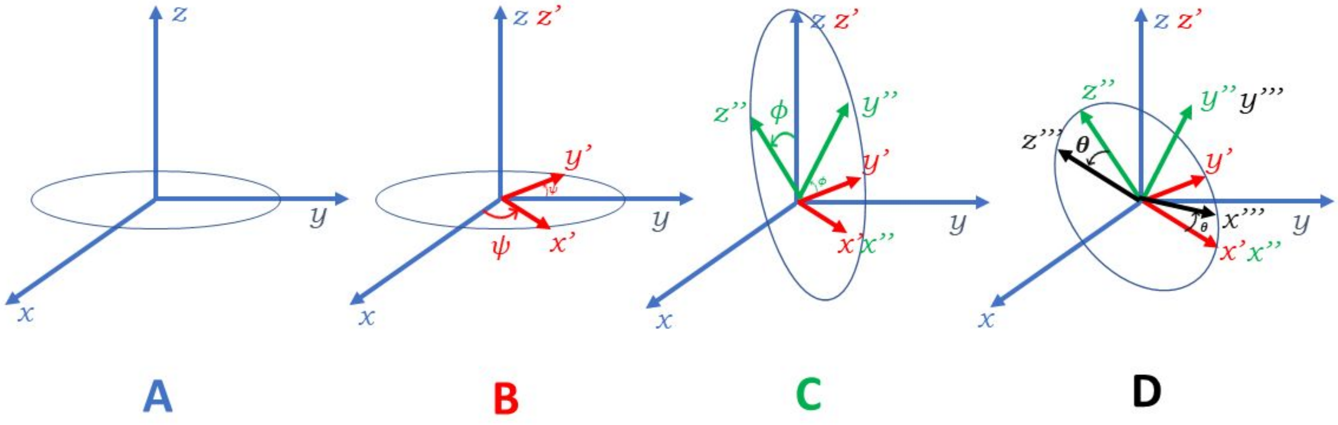


$$
\begin{aligned}
b_1 &= R_{11}a_1 + R_{12}a_2 + R_{13}a_3 \\
b_2 &= R_{21}a_1 + R_{22}a_2 + R_{23}a_3 \\
b_3 &= R_{31}a_1 + R_{32}a_2 + R_{33}a_3
\end{aligned}
$$

$$
{}^{A}R_{B} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}
$$

$$
Q' = {}^{A}R_{B} Q_{A}
$$

Euler Angles



$$^{A}R_{D} = {}^{A}R_{B} \times {}^{B}R_{C} \times {}^{C}R_{D}$$

$$^{A}R_{D} = rot(z,\psi) \times rot(x,\phi) \times rot(y,\theta)$$

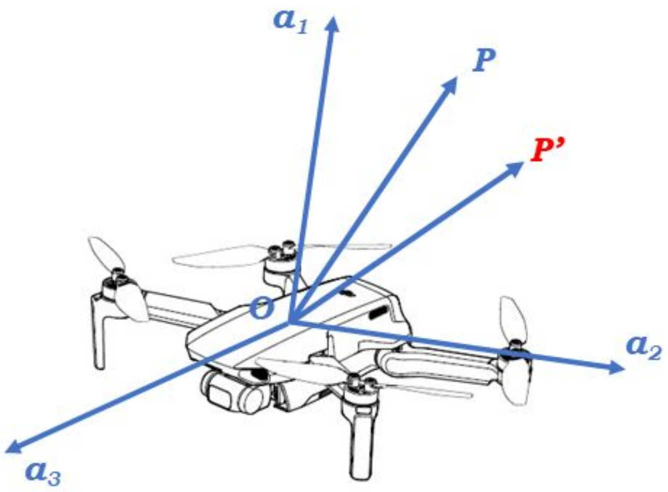
$$rot(z,\psi) = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \; rot(x,\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}, \; rot(y,\theta) = \begin{bmatrix} c\phi & 0 & s\phi \\ 0 & 1 & 0 \\ -s\phi & 0 & c\phi \end{bmatrix}$$

Angular Velocity Vectors

For continuous motion $\dot{q}(t) = \dot{R}(t)p$

$$\dot{q} = \underbrace{\dot{R}(t)R^T(t)}_{\hat{\omega}_s}q$$

$$R^T(t)\dot{q}(t) = \underbrace{R^T(t)\dot{R}(t)}_{\hat{\omega}_b}p$$



Rotation about z-axis

$$rot(z,\psi) = R = \begin{bmatrix} \mathbf{c}\psi & -\mathbf{s}\psi & 0 \\ \mathbf{s}\psi & \mathbf{c}\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^T(t)\dot{R}(t) = \dot{R}(t)R^T(t) = \underbrace{\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\omega}\dot{\psi}.$$

$$\hat{\omega_b} = \hat{\omega_s} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\hat{\omega}}\dot{\psi}$$

$$\overrightarrow{OP} = p_1 a_1 + p_2 a_2 + p_3 a_3$$

$$\overrightarrow{OP'} = q_1 a_1 + q_2 a_2 + q_3 a_3$$

$$\underbrace{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}}_{q} = {}^A R_X \underbrace{\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}}_{p}$$

Thrust and Torque

Thrust $F = k_F \omega^2$ — k_F: fixed parameter; w: angular velocity (RPM)

Rolling Torque $\tau_{roll} = d(F_4 - F_2)$
$= l k_F (w_4^2 - w_2^2)$

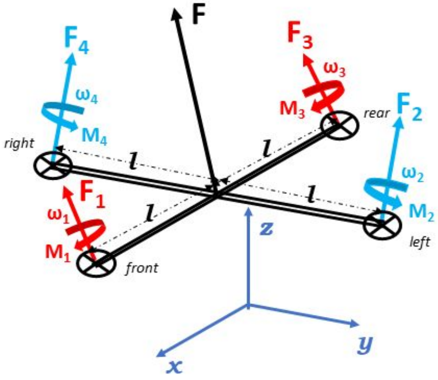
Pitching Torque $\tau_{pitch} = l k_F (w_3^2 - w_1^2)$

Aerodynamic drag $M = k_M w^2$ — k_M: fixed parameter

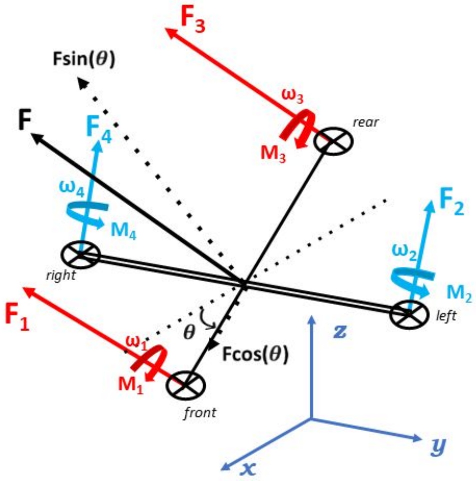
Yaw Torque $\tau_{yaw} = M_1 - M_2 + M_3 - M_4$
$= k_M (w_1^2 - w_2^2 + w_3^2 - w_4^2)$

$$\begin{bmatrix} F \\ \tau \end{bmatrix} = \begin{bmatrix} -k_F & -k_F & -k_F & -k_F \\ 0 & -lk_F & 0 & lk_F \\ -lk_F & 0 & lk_F & 0 \\ k_M & -k_M & k_M & -k_M \end{bmatrix} \begin{bmatrix} w_1^2 \\ w_2^2 \\ w_3^2 \\ w_4^2 \end{bmatrix} = \mathbb{A} \begin{bmatrix} w_1^2 \\ w_2^2 \\ w_3^2 \\ w_4^2 \end{bmatrix} = \mathbb{A}\omega^2$$

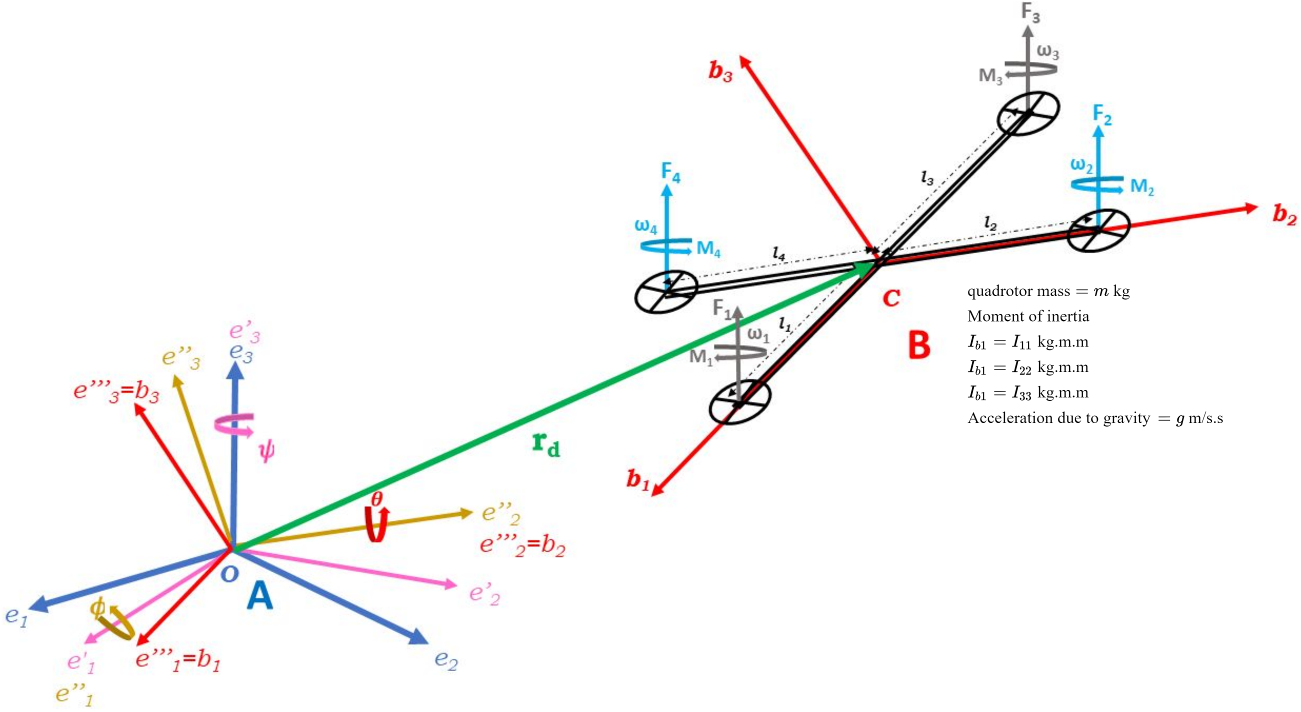
$$\omega = \left(\mathbb{A}^{-1} \begin{bmatrix} F \\ \tau \end{bmatrix} \right)^{\frac{1}{2}} \text{ if } l, k_F, k_M > 0$$



Hover



Forward Flight



quadrotor mass $= m$ kg

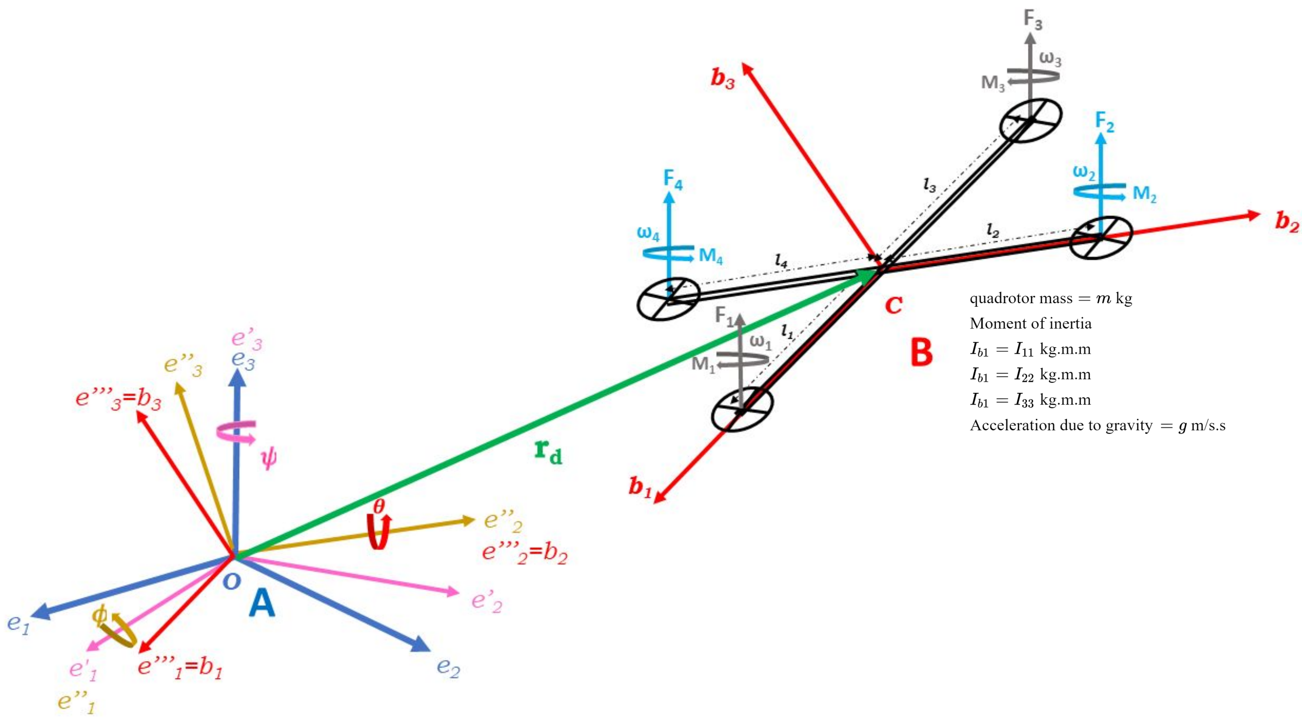
Moment of inertia

$I_{b1} = I_{11}$ kg.m.m

$I_{b1} = I_{22}$ kg.m.m

$I_{b1} = I_{33}$ kg.m.m

Acceleration due to gravity $= g$ m/s.s



F3
ω3
M3
b3
F2
ω2
M2
b2
l3
l2
F4
ω4
M4
l4
C
B
F1
ω1
M1
l1
b1
rd
e'3
e3
e''3
e'''3=b3
ψ
θ
e''2
e'''2=b2
O
A
e1
ϕ
e'1
e''1
e'''1=b1
e'2
e2
quadrotor mass = m kg
Moment of inertia
Ib1 = I11 kg.m.m
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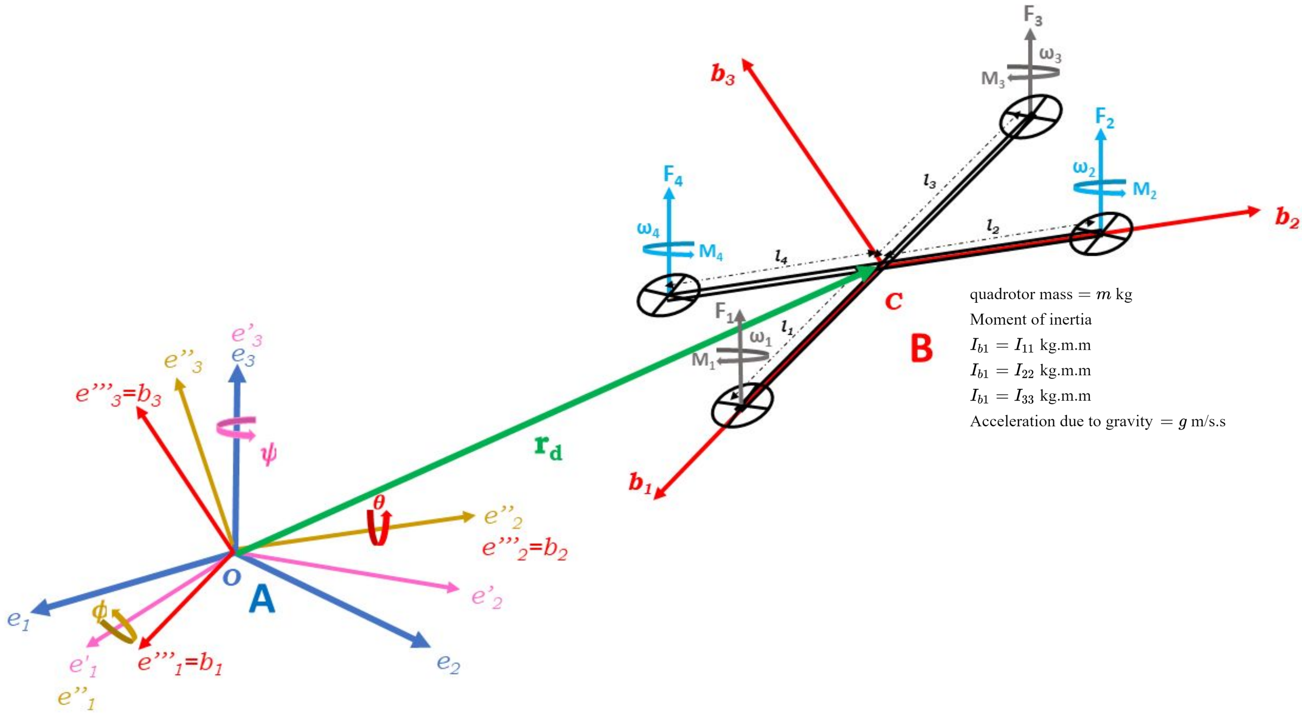
Ib1 = I33 kg.m.m
Acceleration due to gravity = g m/s.s

$$^{A}R_{B} = rot(z,\psi) \times rot(x,\phi) \times rot(y,\theta)$$

$$^{A}R_{B} = \begin{bmatrix} c\psi c\theta - s\phi s\psi s\theta & -c\phi s\psi & c\psi s\theta + c\theta s\phi s\psi \\ c\theta s\psi + c\psi s\phi s\theta & c\phi c\psi & s\psi s\theta - c\psi c\theta s\phi \\ -c\phi s\theta & s\phi & c\phi c\theta \end{bmatrix}$$



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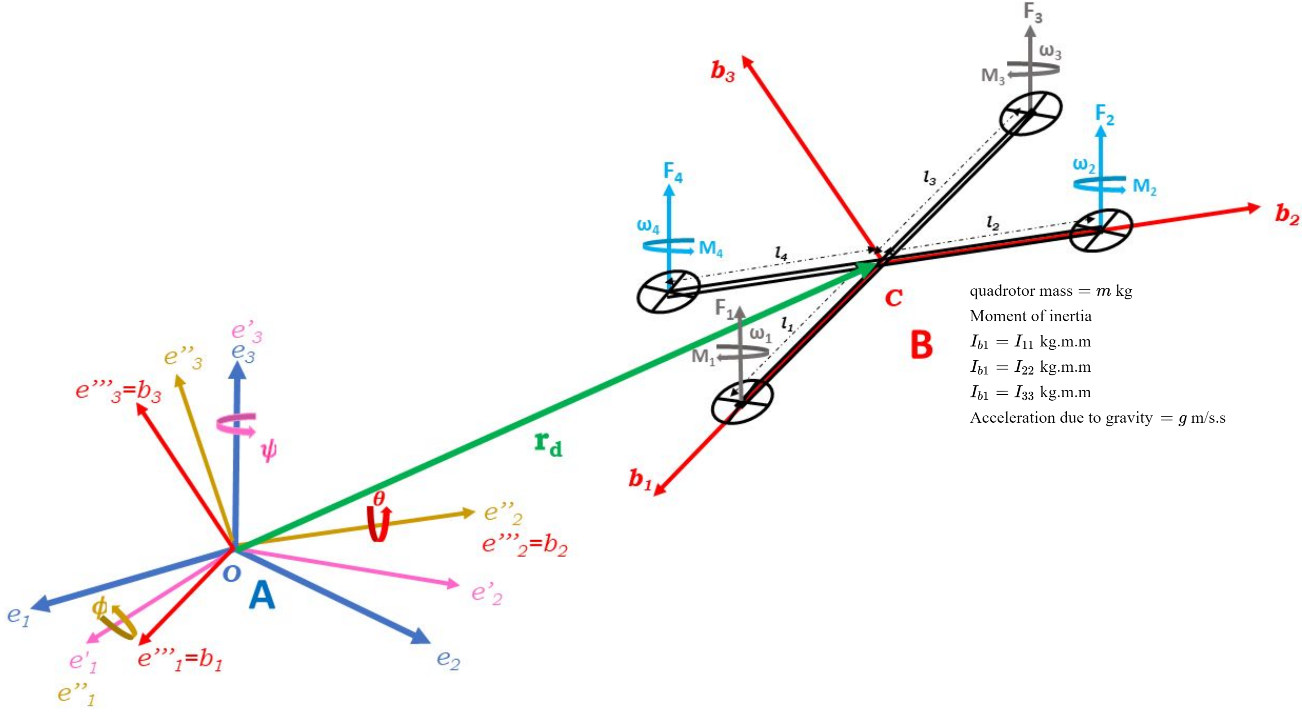
Net force on the quadrotor frame

$$F = F_1 + F_2 + F_3 + F_4 - mge_3$$



Quadrotor dynamics - Translational Motion

$$^{A}R_{B} = rot(z,\psi) \times rot(x,\phi) \times rot(y,\theta)$$

$$^{A}R_{B} = \begin{bmatrix} c\psi c\theta - s\phi s\psi s\theta & -c\phi s\psi & c\psi s\theta + c\theta s\phi s\psi \\ c\theta s\psi + c\psi s\phi s\theta & c\phi c\psi & s\psi s\theta - c\psi c\theta s\phi \\ -c\phi s\theta & s\phi & c\phi c\theta \end{bmatrix}$$

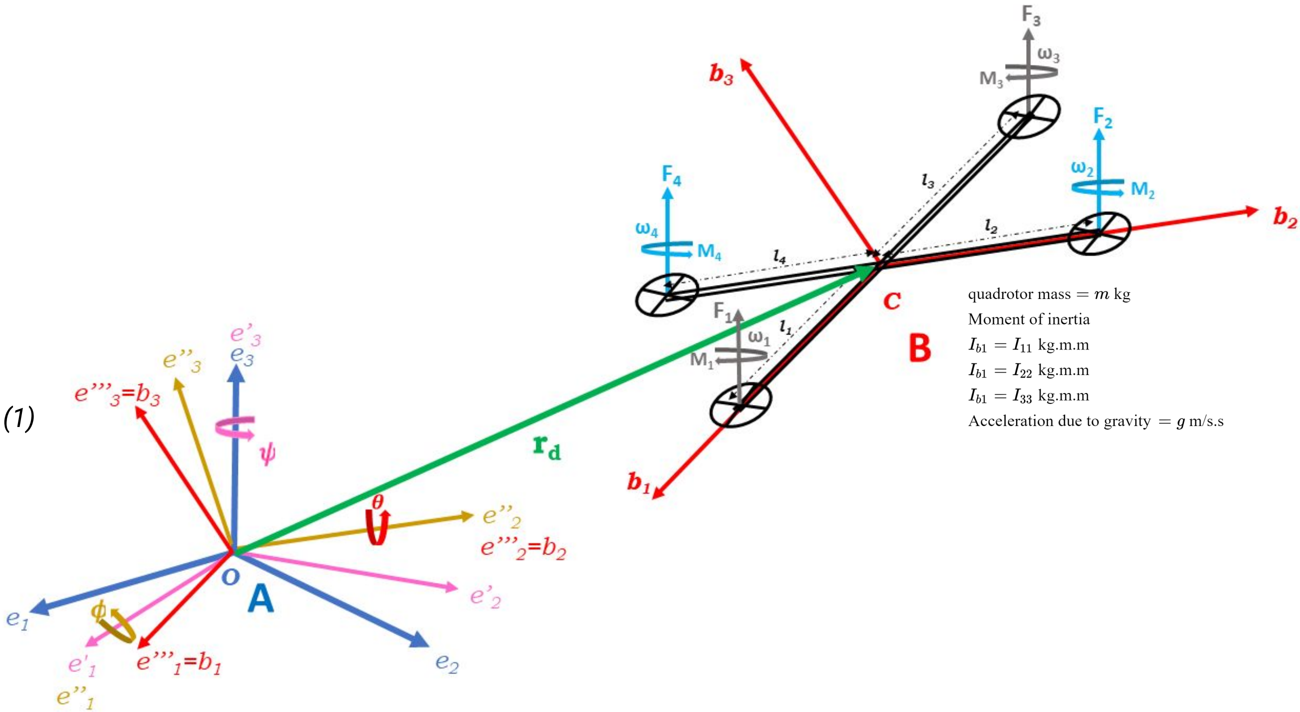
Net force on the quadrotor frame

$$F = F_1 + F_2 + F_3 + F_4 - mge_3$$

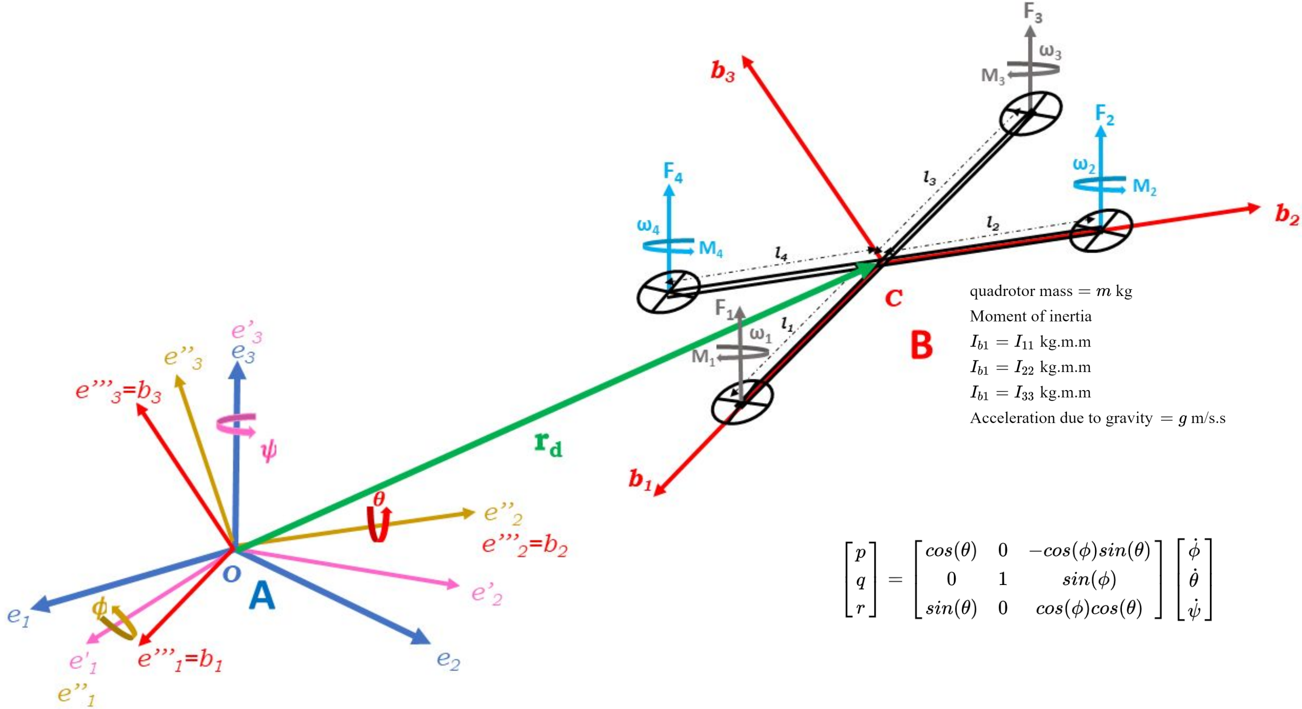
Using Newton's Second Law of motion

$$\ddot{r_d} = \frac{1}{m}\left(\begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + {}^{A}R_{B} \begin{bmatrix} 0 \\ 0 \\ (F_1 + F_2 + F_3 + F_4) \end{bmatrix} \right) \quad (1)$$

TRANSLATIONAL DYNAMICS



Angular velocity : $^{A}\Omega^{B} = pb_1 + qb_2 + rb_3$



quadrotor mass $= m$ kg

Moment of inertia

$I_{b1} = I_{11}$ kg.m.m

$I_{b1} = I_{22}$ kg.m.m

$I_{b1} = I_{33}$ kg.m.m

Acceleration due to gravity $= g$ m/s.s

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} cos(\theta) & 0 & -cos(\phi)sin(\theta) \\ 0 & 1 & sin(\phi) \\ sin(\theta) & 0 & cos(\phi)cos(\theta) \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

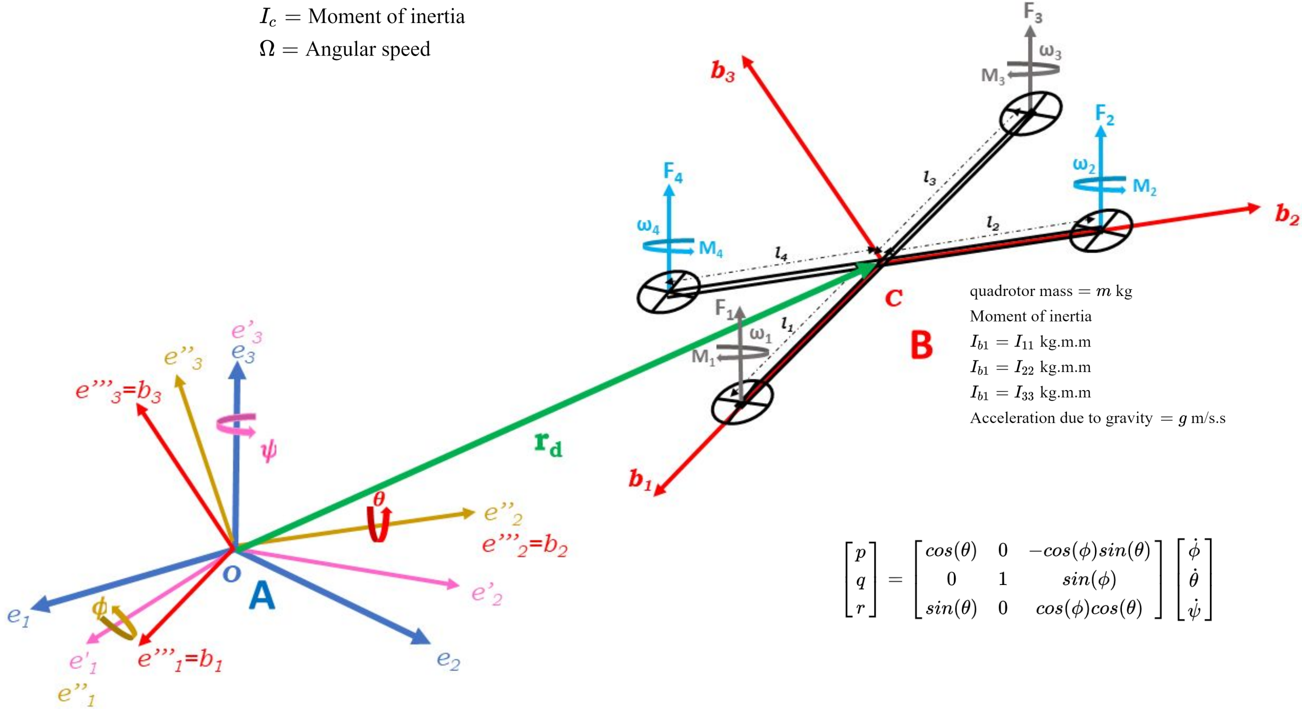
Power Model of a quadrotor

Quadrotor dynamics - Rotational motion

Angular velocity : $^{A}\Omega^{B} = pb_1 + qb_2 + rb_3$

Using Euler's equations $\frac{^{A}d\ H_C}{dt} = M_C$ where $H_C = I_c\Omega$

H_c = angular momentum
M_c = Net moment
I_c = Moment of inertia
Ω = Angular speed



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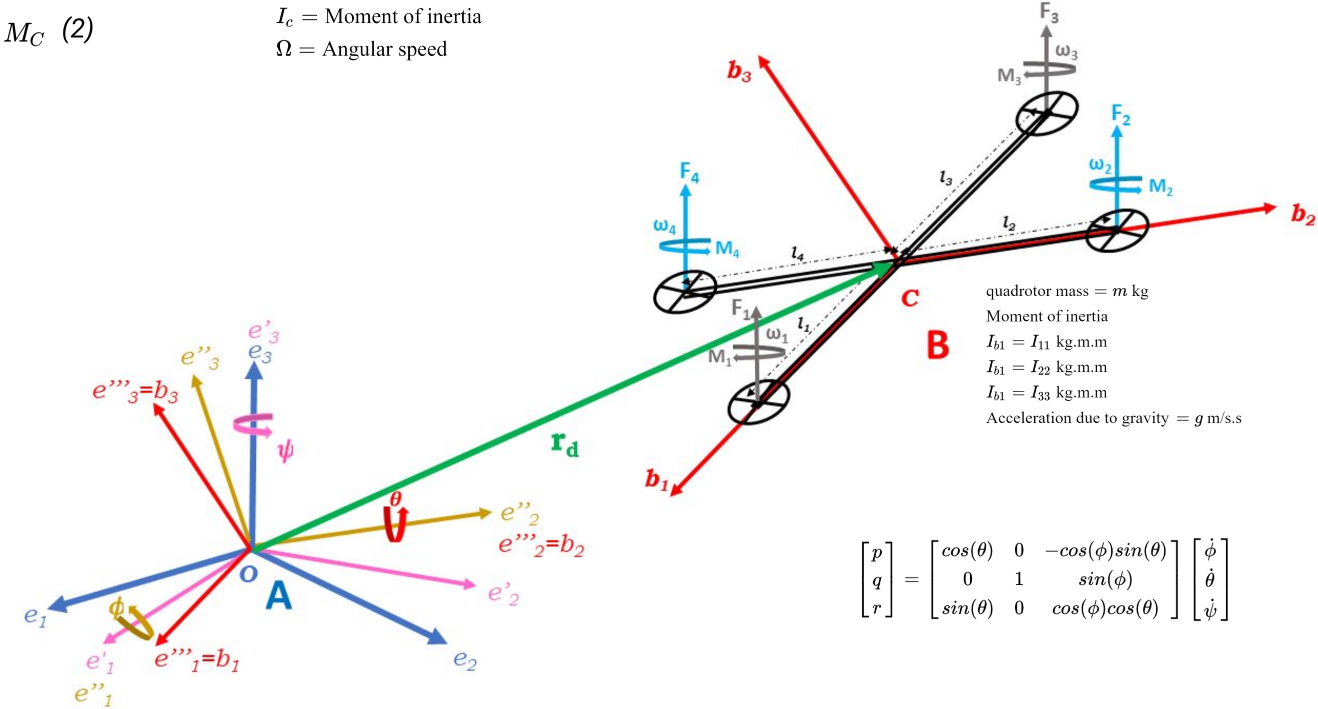
$$\frac{{}^{B}d\ H_C}{dt} + {}^{A}\Omega^{B} \times H_C = M_C \quad (2)$$

$H_c =$ angular momentum
$M_c =$ Net moment
$I_c =$ Moment of inertia
$\Omega =$ Angular speed



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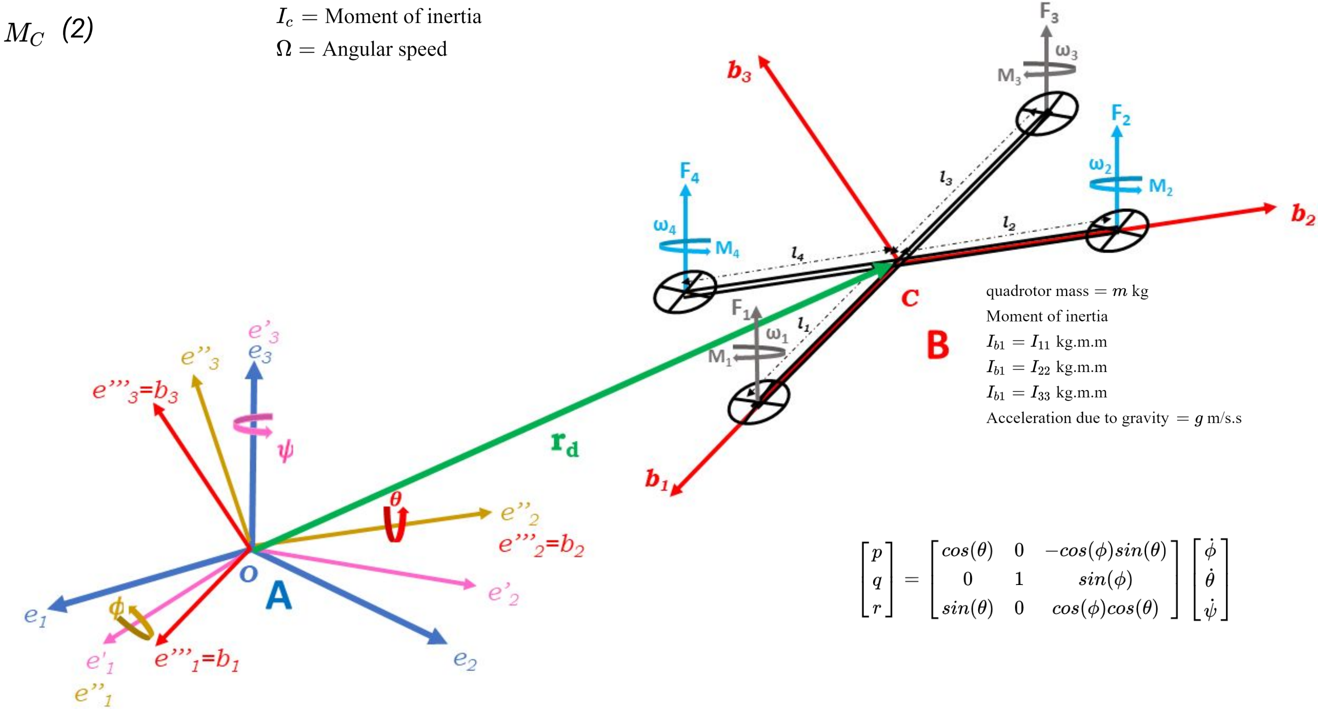
Angular velocity : $^{A}\Omega^{B} = pb_1 + qb_2 + rb_3$

Using Euler's equations $\frac{^{A}d\ H_C}{dt} = M_C$ where $H_C = I_c\Omega$

$$\frac{^{B}d\ H_C}{dt} + {^{A}\Omega^{B}} \times H_C = M_C \quad (2)$$

$$\frac{^{B}d\ H_C}{dt} = I_{11}\dot{p}b_1 + I_{22}\dot{q}b_2 + I_{33}\dot{r}b_3 \quad (3)$$

H_c = angular momentum
M_c = Net moment
I_c = Moment of inertia
Ω = Angular speed



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Power Model of a quadrotor

Quadrotor dynamics - Rotational motion

Angular velocity : ${}^{A}\Omega^{B} = pb_1 + qb_2 + rb_3$

H_c = angular momentum
M_c = Net moment
I_c = Moment of inertia
Ω = Angular speed

Using Euler's equations

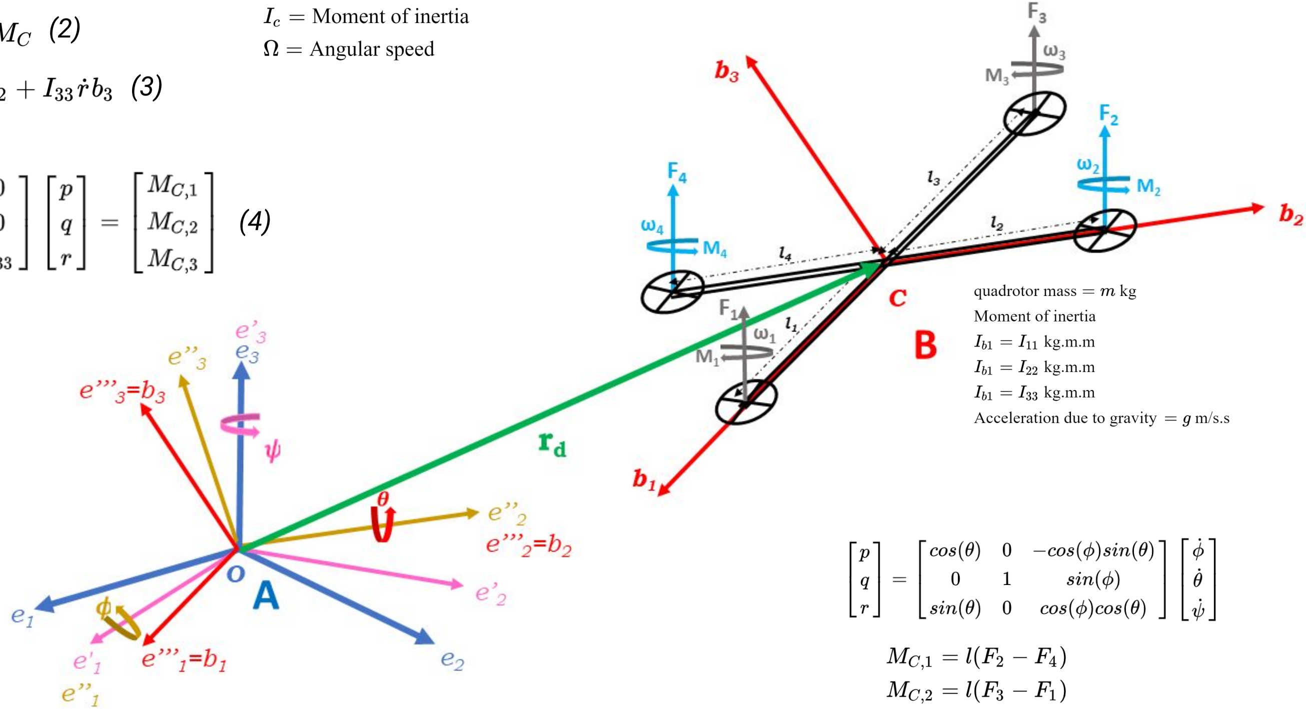
$$\frac{{}^{A}d\ H_C}{dt} = M_C \quad \text{where } H_C = I_c\Omega$$

$$\frac{{}^{B}d\ H_C}{dt} + {}^{A}\Omega^{B} \times H_C = M_C \quad (2)$$

$$\frac{{}^{B}d\ H_C}{dt} = I_{11}\dot{p}b_1 + I_{22}\dot{q}b_2 + I_{33}\dot{r}b_3 \quad (3)$$

Using (3) in (2)

$$\begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} M_{C,1} \\ M_{C,2} \\ M_{C,3} \end{bmatrix} \quad (4)$$



quadrotor mass $= m$ kg
Moment of inertia
$I_{b1} = I_{11}$ kg.m.m
$I_{b1} = I_{22}$ kg.m.m
$I_{b1} = I_{33}$ kg.m.m
Acceleration due to gravity $= g$ m/s.s

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} cos(\theta) & 0 & -cos(\phi)sin(\theta) \\ 0 & 1 & sin(\phi) \\ sin(\theta) & 0 & cos(\phi)cos(\theta) \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$M_{C,1} = l(F_2 - F_4)$$
$$M_{C,2} = l(F_3 - F_1)$$
$$M_{C,3} = M_1 - M_2 + M_3 - M_4$$

Power Model of a quadrotor

Quadrotor dynamics - Rotational motion

Angular velocity : ${}^{A}\Omega^{B} = pb_1 + qb_2 + rb_3$

Using Euler's equations

$$\frac{{}^{A}d\ H_C}{dt} = M_C \quad \text{where } H_C = I_c\Omega$$

$$\frac{{}^{B}d\ H_C}{dt} + {}^{A}\Omega^{B} \times H_C = M_C \quad (2)$$

$$\frac{{}^{B}d\ H_C}{dt} = I_{11}\dot{p}b_1 + I_{22}\dot{q}b_2 + I_{33}\dot{r}b_3 \quad (3)$$

H_c = angular momentum
M_c = Net moment
I_c = Moment of inertia
Ω = Angular speed

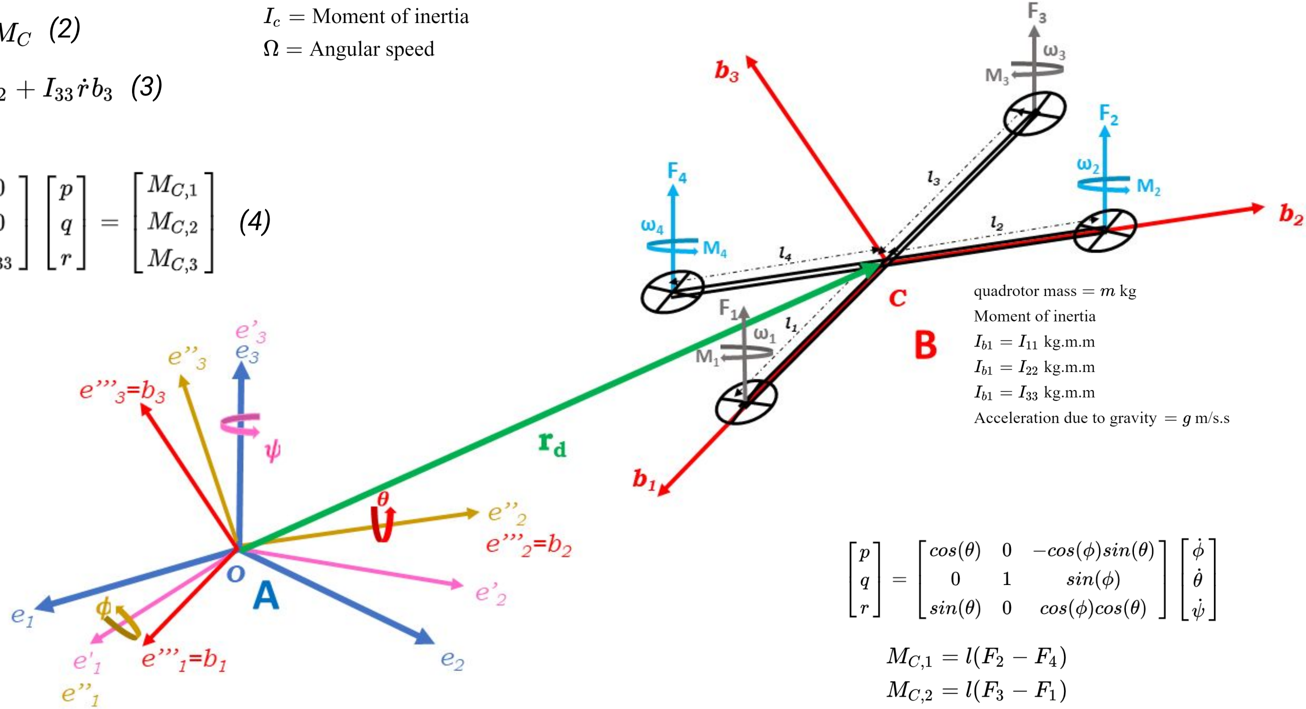
Using (2) and (3)

$$\begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} M_{C,1} \\ M_{C,2} \\ M_{C,3} \end{bmatrix} \quad (4)$$

from (4)

$$\mathbb{I}\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} l(F_2 - F_4) \\ l(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \mathbb{I}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\mathbb{I} = \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix}$$



quadrotor mass $= m$ kg
Moment of inertia
$I_{b1} = I_{11}$ kg.m.m
$I_{b1} = I_{22}$ kg.m.m
$I_{b1} = I_{33}$ kg.m.m
Acceleration due to gravity $= g$ m/s.s

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} cos(\theta) & 0 & -cos(\phi)sin(\theta) \\ 0 & 1 & sin(\phi) \\ sin(\theta) & 0 & cos(\phi)cos(\theta) \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$M_{C,1} = l(F_2 - F_4)$$
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Power Model of a quadrotor

Quadrotor dynamics - Rotational motion

Angular velocity : $^{A}\Omega^{B} = pb_1 + qb_2 + rb_3$

Using Euler's equations

$$\frac{^{A}d\ H_C}{dt} = M_C \quad \text{where } H_C = I_c\Omega$$

$$\frac{^{B}d\ H_C}{dt} + {^{A}\Omega^{B}} \times H_C = M_C \quad (2)$$

$$\frac{^{B}d\ H_C}{dt} = I_{11}\dot{p}b_1 + I_{22}\dot{q}b_2 + I_{33}\dot{r}b_3 \quad (3)$$

$H_c =$ angular momentum
$M_c =$ Net moment
$I_c =$ Moment of inertia
$\Omega =$ Angular speed

Using (2) and (3)

$$\begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} M_{C,1} \\ M_{C,2} \\ M_{C,3} \end{bmatrix} \quad (4)$$

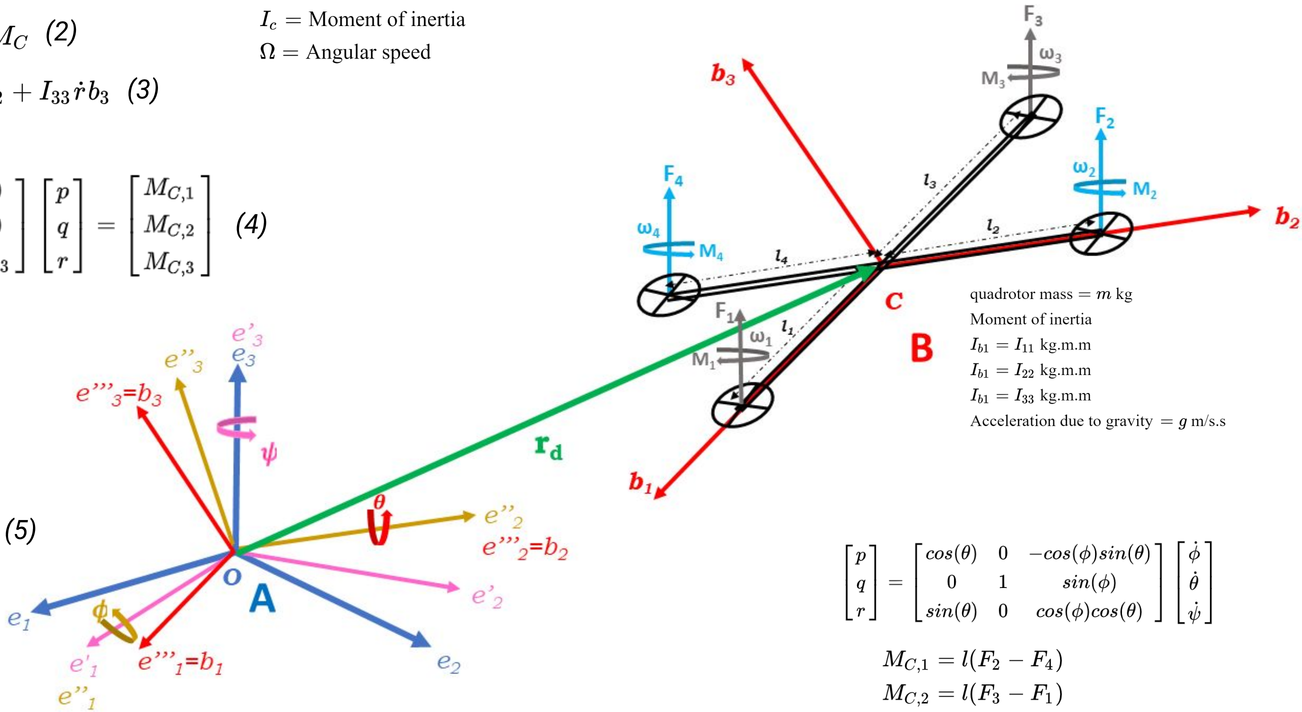
from (4)

$$\mathbb{I} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} l(F_2 - F_4) \\ l(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \mathbb{I} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\mathbb{I} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & l & 0 & -l \\ -l & 0 & l & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \mathbb{I} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (5)$$

ROTATIONAL DYNAMICS

$$\mathbb{I} = \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix} \qquad \gamma = \frac{k_M}{k_F}$$



quadrotor mass $= m$ kg
Moment of inertia
$I_{b1} = I_{11}$ kg.m.m
$I_{b1} = I_{22}$ kg.m.m
$I_{b1} = I_{33}$ kg.m.m
Acceleration due to gravity $= g$ m/s.s

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} cos(\theta) & 0 & -cos(\phi)sin(\theta) \\ 0 & 1 & sin(\phi) \\ sin(\theta) & 0 & cos(\phi)cos(\theta) \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$M_{C,1} = l(F_2 - F_4)$$
$$M_{C,2} = l(F_3 - F_1)$$
$$M_{C,3} = M_1 - M_2 + M_3 - M_4$$

Power Model of a quadrotor

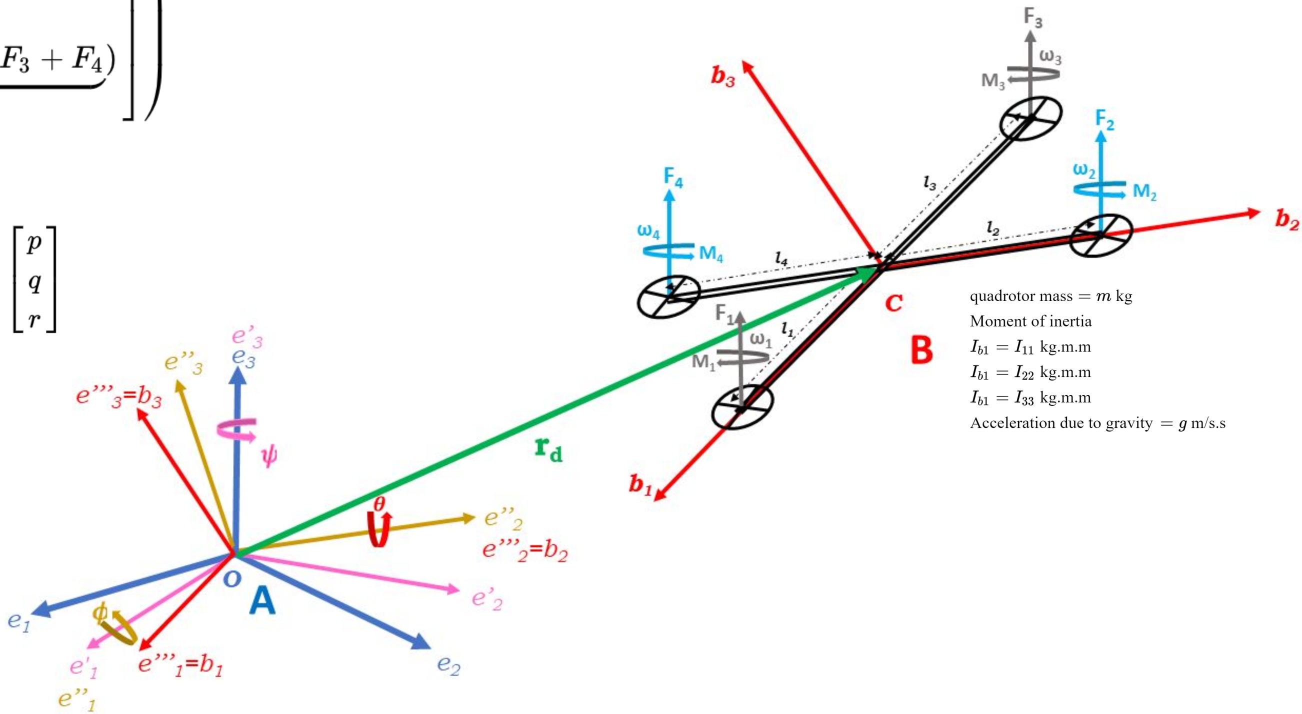
Quadrotor dynamics

$$\begin{bmatrix} \ddot{r_1} \\ \ddot{r_2} \\ \ddot{r_3} \end{bmatrix} = \frac{1}{m}\left(\begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + {}^{A}R_B \begin{bmatrix} 0 \\ 0 \\ \underbrace{(F_1 + F_2 + F_3 + F_4)}_{u_1} \end{bmatrix} \right)$$

TRANSLATIONAL DYNAMICS

$$\mathbb{I} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & l & 0 & -l \\ -l & 0 & l & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}}_{u_2} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \mathbb{I} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

ROTATIONAL DYNAMICS



quadrotor mass $= m$ kg

Moment of inertia

$I_{b1} = I_{11}$ kg.m.m

$I_{b1} = I_{22}$ kg.m.m

$I_{b1} = I_{33}$ kg.m.m

Acceleration due to gravity $= g$ m/s.s

Desired trajectory

$$r^{des}(t), \dot{r}^{des}(t), \ddot{r}^{des}(t)$$

$$\psi^{des}(t), \dot{\psi}^{des}(t), \ddot{\psi}^{des}(t)$$

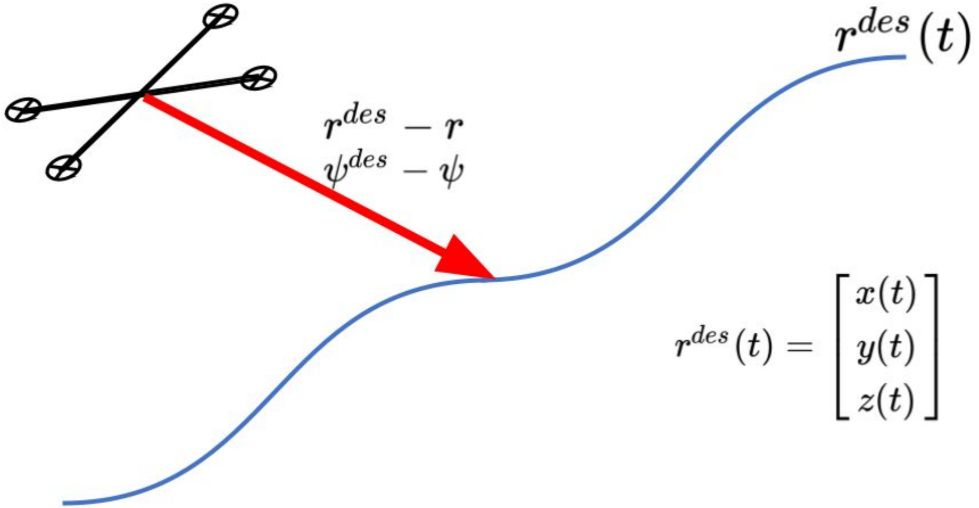
Error Dynamics

$$e_p = r^{des}(t) - r(t)$$

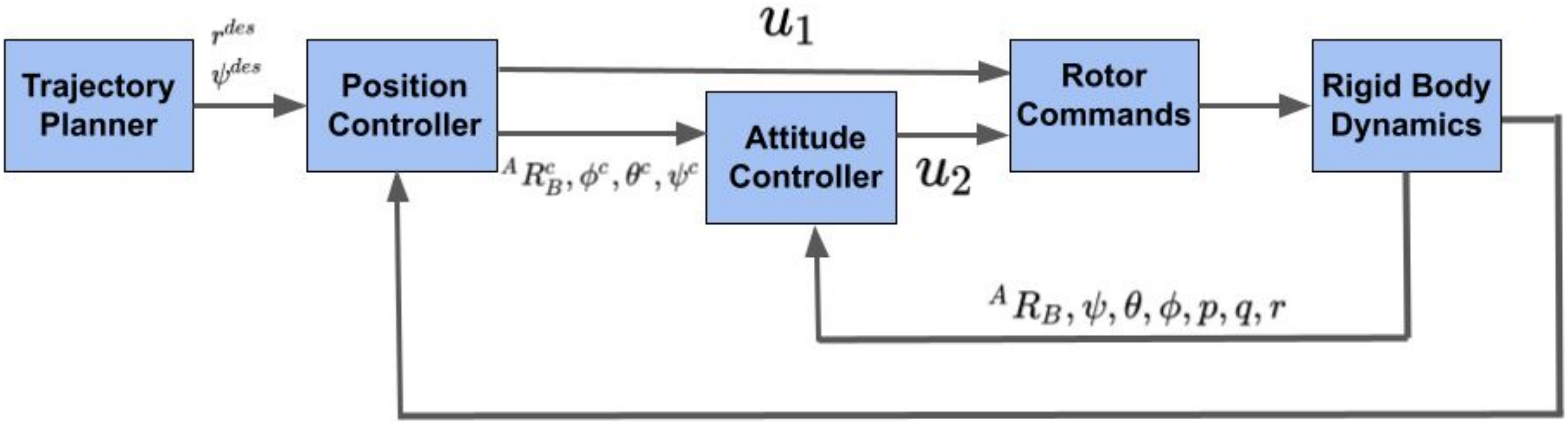
$$e_v = \dot{r}^{des}(t) - \dot{r}(t)$$

For exponential decay of error

$$\ddot{r}^{des}(t) - \ddot{r}^{c}(t) + K_d e_v + K_p e_p = 0 \quad (6)$$



Control Architecture



Linearized dynamics around hover

$$\boldsymbol{r}^{des}(\boldsymbol{t}) = \boldsymbol{r} = \boldsymbol{r}_0, \theta = \phi = 0, \psi = \psi_0$$

$$\dot{\boldsymbol{r}} = 0, \dot{\phi} = \dot{\theta} = \dot{\psi} = 0$$

$$(cos\phi \approx 1, cos\theta \approx 1, sin\phi \approx \phi, sin\theta \approx \theta)$$

$$\ddot{r}_1 = g\left(\theta \cos\psi_0 + \phi \sin\psi_0\right)$$
$$\ddot{r}_2 = g\left(\theta \sin\psi_0 - \phi \cos\psi_0\right)$$
$$\ddot{r}_3 = \frac{1}{m}\underbrace{(F_1 + F_2 + F_3 + F_4)}_{u_1} - g$$

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \mathbb{I}^{-1} \underbrace{\begin{bmatrix} 0 & l & 0 & -l \\ -l & 0 & l & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}}_{u_2}$$

$$p = \dot{\phi}, q = \dot{\theta}$$

$$\ddot{r_d} = \frac{1}{m}\left(\begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + {}^{A}R_B \begin{bmatrix} 0 \\ 0 \\ \underbrace{(F_1 + F_2 + F_3 + F_4)}_{u_1} \end{bmatrix}\right)$$

TRANSLATIONAL DYNAMICS

$$\mathbb{I} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & l & 0 & -l \\ -l & 0 & l & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}}_{u_2} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \mathbb{I} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

ROTATIONAL DYNAMICS

Power Model of a quadrotor

Quadrotor control

$$\ddot{r}_1 = g\left(\theta \cos\psi_0 + \phi \sin\psi_0\right) \quad (7)$$
$$\ddot{r}_2 = g\left(\theta \sin\psi_0 - \phi \cos\psi_0\right) \quad (8)$$
$$\ddot{r}_3 = \frac{1}{m}\underbrace{\left(F_1 + F_2 + F_3 + F_4\right)}_{u_1} - g \quad (9)$$

LINEARIZED TRANSLATIONAL DYNAMICS

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \mathbb{I}^{-1} \underbrace{\begin{bmatrix} 0 & l & 0 & -l \\ -l & 0 & l & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}}_{u_2}$$

LINEARIZED ROTATIONAL DYNAMICS

from (6) which is the position control law

$$\left(\ddot{r}_i^{des} - \ddot{r}_i^{c}\right) + k_{d,i}\left(\dot{r}_i^{des} - \dot{r}_i\right) + k_{p,i}\left(r_i^{des} - r_i\right) = 0 \quad (10)$$

(10) rewritten as

$$\ddot{r}_i^{c} = \ddot{r}_i^{des} + k_{d,i}\left(\dot{r}_i^{des} - \dot{r}_i\right) + k_{p,i}\left(r_i^{des} - r_i\right)$$

Putting (10) in (9)

$$u_1 = mg + m\ddot{r}_3^{c} = mg - m\left(k_{d,3}\dot{r}_3 + k_{p,3}\left(r_3 - r_{3,0}\right)\right) \quad (11)$$

CONTROL LAW FOR POSITION

Using (7) and (8), we find commanded θ_c, ϕ_c

$$\phi^c = \frac{1}{g}\left(\ddot{r}_1^{\,c} \sin\psi_0 - \ddot{r}_2^{\,c} \cos\psi_0\right) \quad (12)$$
$$\theta^c = \frac{1}{g}\left(\ddot{r}_1^{\,c} \cos\psi_0 + \ddot{r}_2^{\,c} \sin\psi_0\right) \quad (13)$$

$$\ddot{r}_1^{c} = \ddot{r}_1^{des} + k_{d,1}\left(\dot{r}_1^{des} - \dot{r}_1\right) + k_{p,1}\left(r_1^{des} - r_1\right)$$
$$\ddot{r}_2^{c} = \ddot{r}_2^{des} + k_{d,2}\left(\dot{r}_2^{des} - \dot{r}_1\right) + k_{p,2}\left(r_2^{des} - r_2\right)$$

Using (12) and (13), we define

$$\boldsymbol{u}_2 = \begin{bmatrix} k_{p,\phi}\left(\phi^c - \phi\right) + k_{d,\phi}\left(p^c - p\right) \\ k_{p,\theta}\left(\theta^c - \theta\right) + k_{d,\theta}\left(q^c - q\right) \\ k_{p,\psi}\left(\psi^c - \psi\right) + k_{d,\psi}\left(r^c - r\right) \end{bmatrix}$$

CONTROL LAW FOR ATTITUDE

$$u_1 = mg + m\ddot{r}_3^c = mg - m\left(k_{d,3}\dot{r}_3 + k_{p,3}\left(r_3 - r_{3,0}\right)\right)$$
$$= F_1 + F_2 + F_3 + F_4 \qquad (14)$$

$$\boldsymbol{u}_2 = \begin{bmatrix} k_{p,\phi}\left(\phi^c - \phi\right) + k_{d,\phi}\left(p^c - p\right) \\ k_{p,\theta}\left(\theta^c - \theta\right) + k_{d,\theta}\left(q^c - q\right) \\ k_{p,\psi}\left(\psi^c - \psi\right) + k_{d,\psi}\left(r^c - r\right) \end{bmatrix} = \begin{bmatrix} 0 & l & 0 & -l \\ -l & 0 & l & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

$$\begin{bmatrix} u_{2,1} \\ u_{2,2} \\ u_{2,3} \end{bmatrix} = \begin{bmatrix} l(F_2 - F_4) \\ l(F_3 - F_1) \\ \gamma(F_1 - F_2 + F_3 - F_4) \end{bmatrix} \begin{matrix} (15) \\ (16) \\ (17) \end{matrix}$$

from (14),(15),(16),(17)

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 & -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 0 & -\frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{2} & 0 & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} u_1 \\ \frac{u_{2,1}}{l} \\ \frac{u_{2,2}}{l} \\ \frac{u_{2,3}}{\gamma} \end{bmatrix}$$

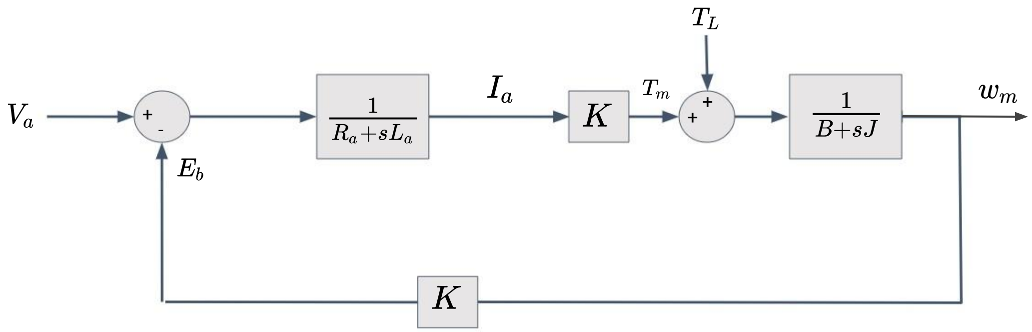
$$\ddot{r}_1 = g\left(\theta\cos\psi_0 + \phi\sin\psi_0\right)$$
$$\ddot{r}_2 = g\left(\theta\sin\psi_0 - \phi\cos\psi_0\right)$$
$$\ddot{r}_3 = \frac{1}{m}\underbrace{\left(F_1 + F_2 + F_3 + F_4\right)}_{u_1} - g$$

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \mathbb{I}^{-1} \underbrace{\begin{bmatrix} 0 & l & 0 & -l \\ -l & 0 & l & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}}_{u_2}$$

Power Model of a quadrotor
Motor Power

Rotor speed profiles

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \sqrt{\frac{1}{k_F} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}} \qquad \text{As } F = k_F \omega^2$$



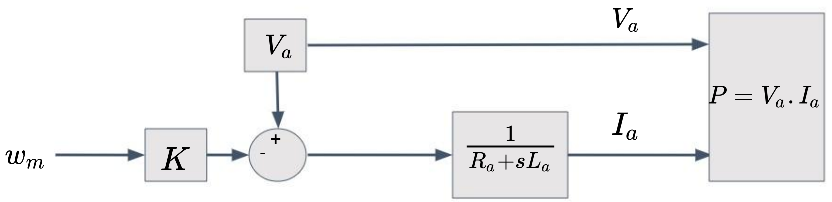
Model of Armature Controlled Separately Excited DC Motor

V_a = Applied Voltage
I_a = Armature Current
w_m = Angular rSpeed
E_b = Back EMF
K = Motor Constant
R_a = Armature Resistance
L_a = Armature Inductance
B = Friction coefficient
J = Moment of Inertia

Power profiles

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = f \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}$$

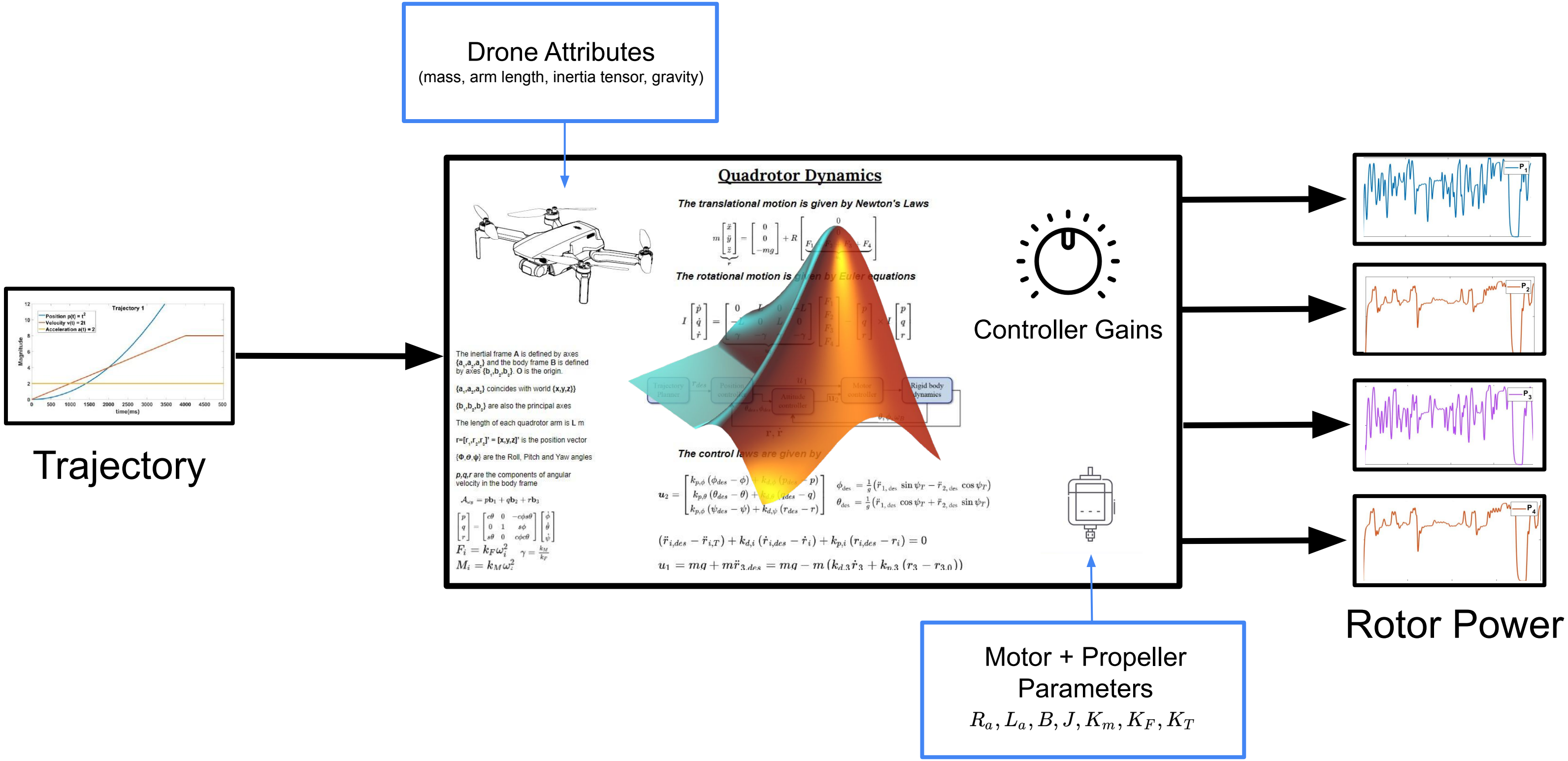
f maps rotor power from rotor speed



Model to get Power from Speed

Power Model of a quadrotor

Power Model design using MATLAB



Power Model of a quadrotor

Power Calculations

Quadrotor Attributes $\{m = 0.18kg, L = 0.018m, I = \begin{bmatrix} 0.0025 & 0 & 0 \\ 0 & 0.000232 & 0 \\ 0 & 0 & 0.0003738 \end{bmatrix} kg.m.m\}$

Motor Parameters $\{R_a = 1\Omega, L_a = 1H, J = 5Kg.m.m, B = 0.01N.m.s, K = 1.6V/rad/s, K_F = K_M = 1, v(t) = 1V\}$

Controller Gains

$\{k_{p,x}, k_{p,y}, k_{p,z}, k_{d,x}, k_{d,y}, k_{d,z}\} = \{200, 200, 100, 40, 40, 20\}$

$\{k_{p,\phi}, k_{p,\theta}, k_{p,\psi}, k_{d,\phi}, k_{d,\theta}, k_{d,\psi}\} = \{100, 100, 100, 2, 2, 2\}$

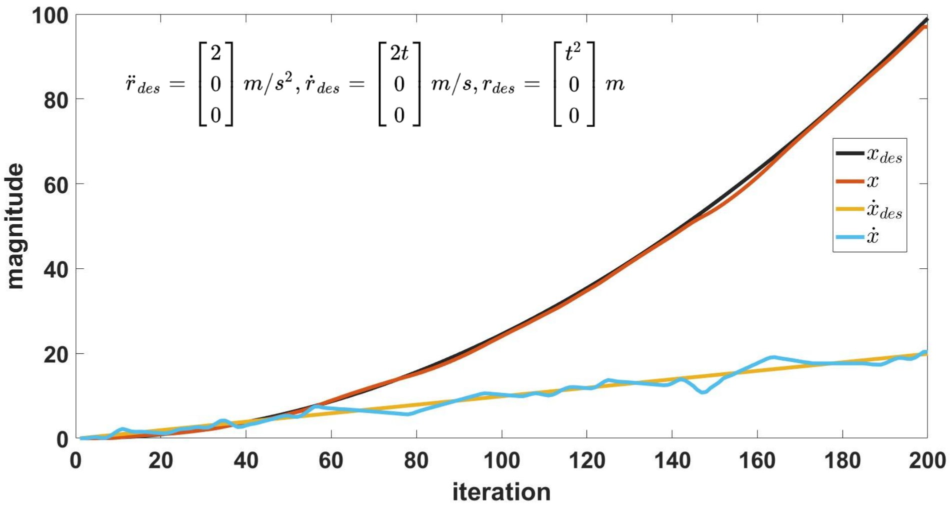
Solver Details

$ode45$

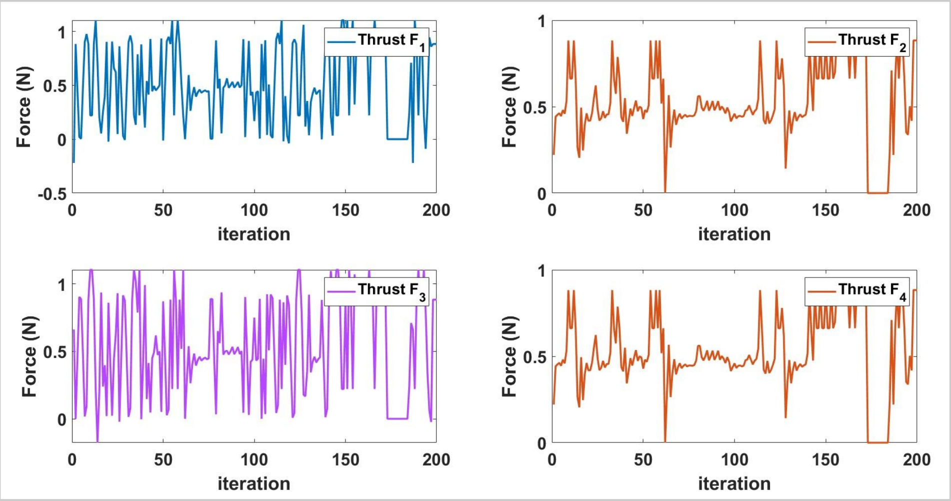
Step size = 0.01s

Span of each iteration = 0.05s

Quadrotor Position



Thrust Profiles



Power Model of a quadrotor

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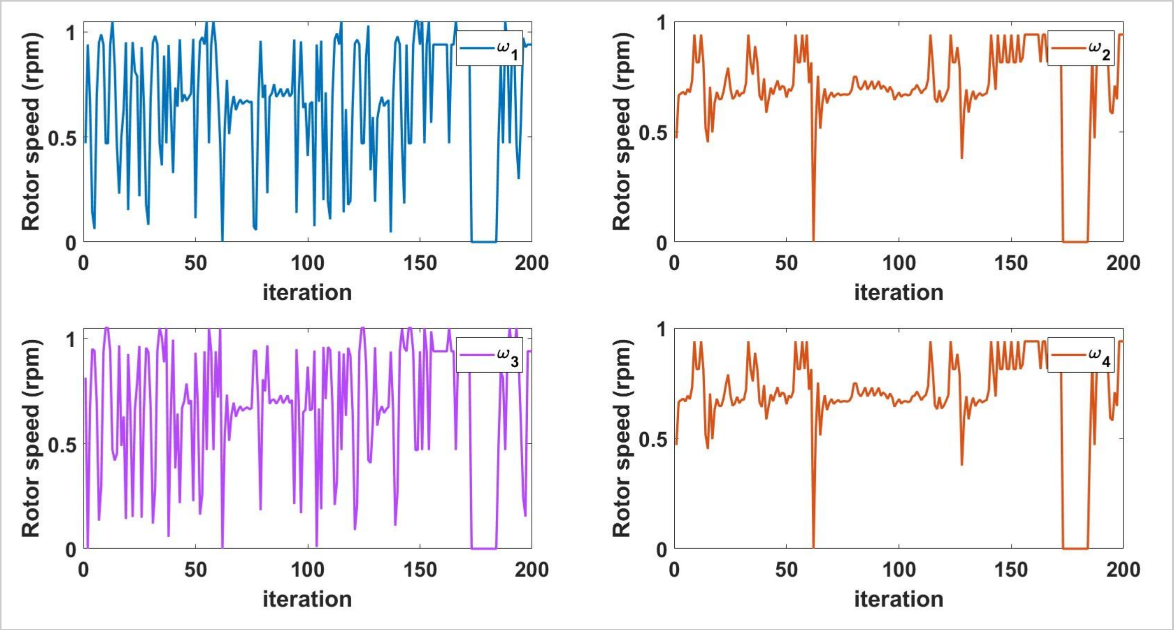
Motor Parameters $\{R_a = 1\Omega, L_a = 1H, J = 5Kg.m.m, B = 0.01N.m.s, K = 1.6V/rad/s, K_F = K_M = 1, v(t) = 1V\}$

Controller Gains

$\{k_{p,x}, k_{p,y}, k_{p,z}, k_{d,x}, k_{d,y}, k_{d,z}\} = \{200, 200, 100, 40, 40, 20\}$

$\{k_{p,\phi}, k_{p,\theta}, k_{p,\psi}, k_{d,\phi}, k_{d,\theta}, k_{d,\psi}\} = \{100, 100, 100, 2, 2, 2\}$

Rotor Speed Profiles



Power Model of a quadrotor

Power Calculations

Quadrotor Attributes $\{m = 0.18kg, L = 0.018m, I = \begin{bmatrix} 0.0025 & 0 & 0 \\ 0 & 0.000232 & 0 \\ 0 & 0 & 0.0003738 \end{bmatrix} kg.m.m\}$

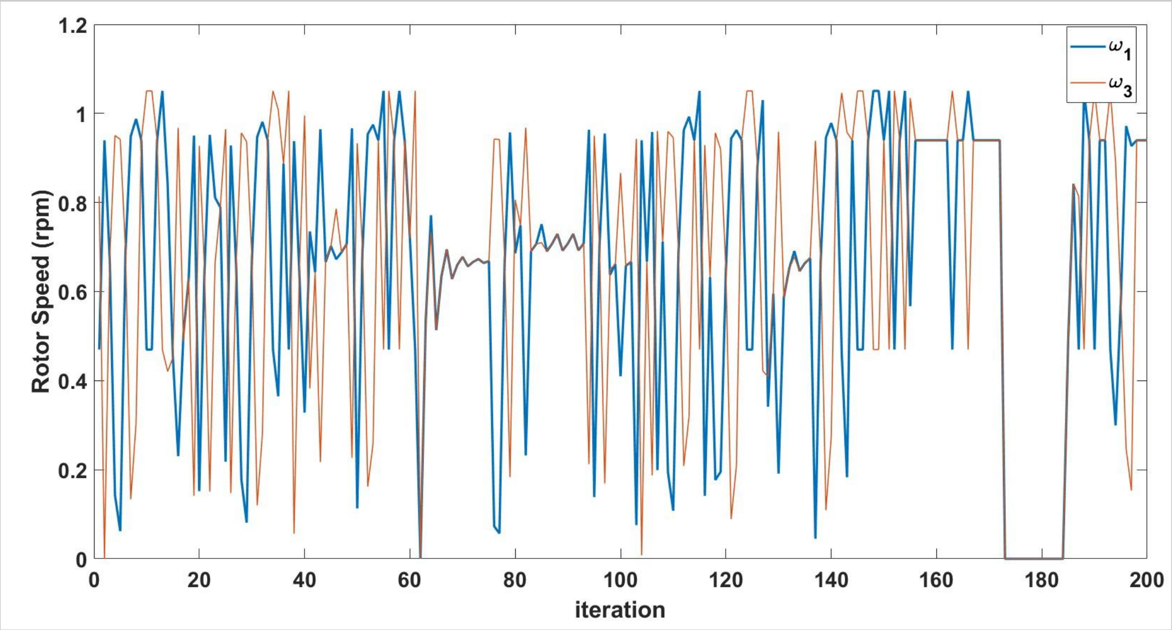
Motor Parameters $\{R_a = 1\Omega, L_a = 1H, J = 5Kg.m.m, B = 0.01N.m.s, K = 1.6V/rad/s, K_F = K_M = 1, v(t) = 1V\}$

Controller Gains

$$\{k_{p,x}, k_{p,y}, k_{p,z}, k_{d,x}, k_{d,y}, k_{d,z}\} = \{200, 200, 100, 40, 40, 20\}$$

$$\{k_{p,\phi}, k_{p,\theta}, k_{p,\psi}, k_{d,\phi}, k_{d,\theta}, k_{d,\psi}\} = \{100, 100, 100, 2, 2, 2\}$$

Rotor Speed Profiles



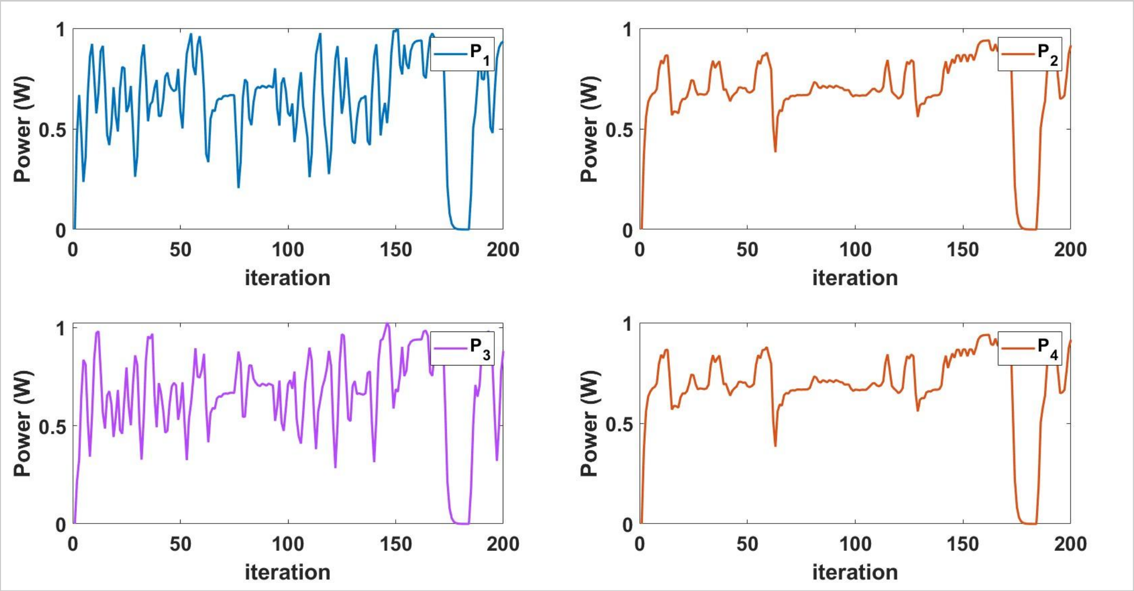
Power Model of a quadrotor
Power Calculations

Quadrotor Attributes $\{m = 0.18kg, L = 0.018m, I = \begin{bmatrix} 0.0025 & 0 & 0 \\ 0 & 0.000232 & 0 \\ 0 & 0 & 0.0003738 \end{bmatrix} kg.m.m\}$

Motor Parameters $\{R_a = 1\Omega, L_a = 1H, J = 5Kg.m.m, B = 0.01N.m.s, K = 1.6V/rad/s, K_F = K_M = 1, v(t) = 1V\}$

Controller Gains
$\{k_{p,x}, k_{p,y}, k_{p,z}, k_{d,x}, k_{d,y}, k_{d,z}\} = \{200, 200, 100, 40, 40, 20\}$
$\{k_{p,\phi}, k_{p,\theta}, k_{p,\psi}, k_{d,\phi}, k_{d,\theta}, k_{d,\psi}\} = \{100, 100, 100, 2, 2, 2\}$

Power Profiles



Power Model of a quadrotor

Power consumption comparison based on maneuver

Quadrotor Attributes $\{m = 0.18kg, L = 0.018m, I = \begin{bmatrix} 0.0025 & 0 & 0 \\ 0 & 0.000232 & 0 \\ 0 & 0 & 0.0003738 \end{bmatrix} kg.m.m\}$

Solver Details
$ode45$
Step size = 0.01s
Span of each iteration = 0.05s

Motor Parameters $\{R_a = 1\Omega, L_a = 1H, J = 5Kg.m.m, B = 0.01N.m.s, K = 1.6V/rad/s, K_F = K_M = 1, v(t) = 1V\}$

Controller Gains

$\{k_{p,x}, k_{p,y}, k_{p,z}, k_{d,x}, k_{d,y}, k_{d,z}\} = \{200, 200, 100, 40, 40, 20\}$

$\{k_{p,\phi}, k_{p,\theta}, k_{p,\psi}, k_{d,\phi}, k_{d,\theta}, k_{d,\psi}\} = \{100, 100, 100, 2, 2, 2\}$

Desired Trajectory

Pitch

$$\ddot{r}_{des} = \begin{bmatrix} 0.375t^3 - 0.00285t^4 + 0.0001t^5 \\ 0 \\ 0 \end{bmatrix} m/s^2, \dot{r}_{des} = \begin{bmatrix} 0.1125t^2 - 0.0112t^3 + 0.0005t^4 \\ 0 \\ 0 \end{bmatrix} m/s, r_{des} = \begin{bmatrix} 0.225t - 0.0336t^2 + 0.002t^3 \\ 0 \\ 0 \end{bmatrix} m$$

Roll

$$\ddot{r}_{des} = \begin{bmatrix} 0 \\ 0.375t^3 - 0.00285t^4 + 0.0001t^5 \\ 0 \end{bmatrix} m/s^2, \dot{r}_{des} = \begin{bmatrix} 0 \\ 0.1125t^2 - 0.0112t^3 + 0.0005t^4 \\ 0 \end{bmatrix} m/s, r_{des} = \begin{bmatrix} 0 \\ 0.225t - 0.0336t^2 + 0.002t^3 \\ 0 \end{bmatrix} m$$

Hover

$$\ddot{r}_{des} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} m/s^2, \dot{r}_{des} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} m/s, r_{des} = \begin{bmatrix} 0 \\ 0 \\ 11 \end{bmatrix} m$$

RMS values of Thrust, Rotor Speed and Power

Maneuver	Thrust (N)				Rotor Speed (RPM)				Rotor Power(W)				Total Power (W)	Difference(%)
	F1	F2	F3	F4	ω1	ω2	ω3	ω4	P1	P2	P3	P4		
Hover	0.4405	0.4405	0.4405	0.4405	0.6594	0.6594	0.6594	0.6594	0.6551	0.6551	0.6551	0.6551	2.6204	-
Pitch	0.5177	0.5181	0.5571	0.5181	0.6994	0.7151	0.7304	0.7151	0.6846	0.7117	0.7203	0.7117	2.8283	8
Roll	0.5063	0.5720	0.5063	0.5442	0.7048	0.7150	0.7048	0.6984	0.7019	0.6977	0.7019	0.6761	2.7776	6

Path Planning

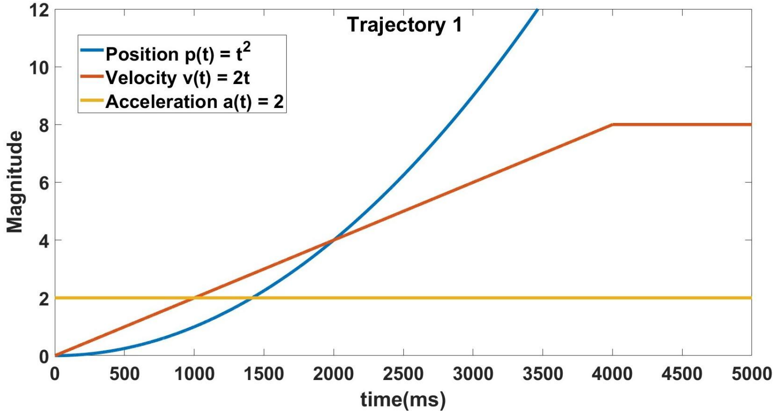
Power consumption comparison based on trajectories

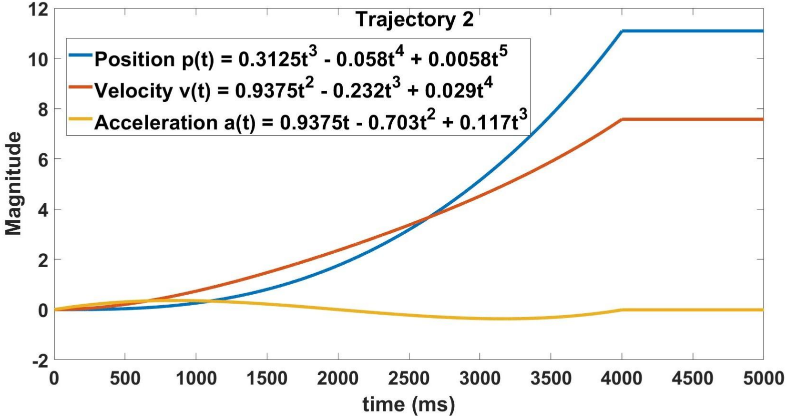
Quadrotor Attributes $\{m = 0.18kg, L = 0.018m, I = \begin{bmatrix} 0.0025 & 0 & 0 \\ 0 & 0.000232 & 0 \\ 0 & 0 & 0.0003738 \end{bmatrix} kg.m.m\}$

Motor Parameters $\{R_a = 1\Omega, L_a = 1H, J = 5Kg.m.m, B = 0.01N.m.s, K = 1.6V/rad/s, K_F = K_M = 1, v(t) = 1V\}$

Controller Gains
$\{k_{p,x}, k_{p,y}, k_{p,z}, k_{d,x}, k_{d,y}, k_{d,z}\} = \{200, 200, 100, 40, 40, 20\}$
$\{k_{p,\phi}, k_{p,\theta}, k_{p,\psi}, k_{d,\phi}, k_{d,\theta}, k_{d,\psi}\} = \{100, 100, 100, 2, 2, 2\}$

Objective of the quadrotor is to reach 11m along the x-axis





Trajectory	Time of convergence (s)	Rotor Power (W)				Total Power (W)
		P1	P2	P3	P4	
Trajectory 1	3.3	0.6695	0.7126	0.6889	0.7126	2.7836
Trajectory 2	4	0.4796	0.6602	0.7493	0.6602	2.5439

Field Experiments using DJI Air 2

Power Data

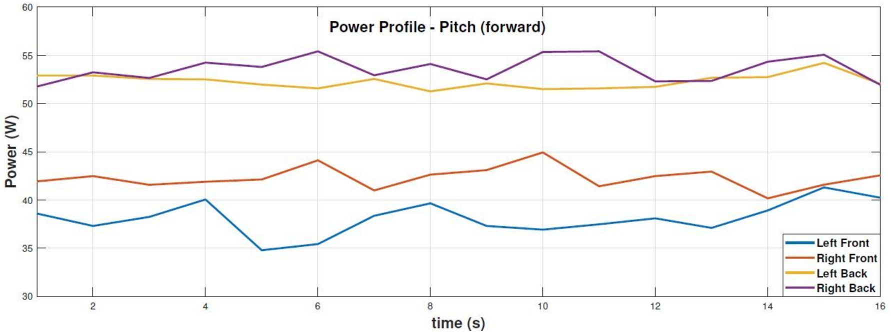


Fig 4 - DJI Air 2 forward flight with a forward pitch

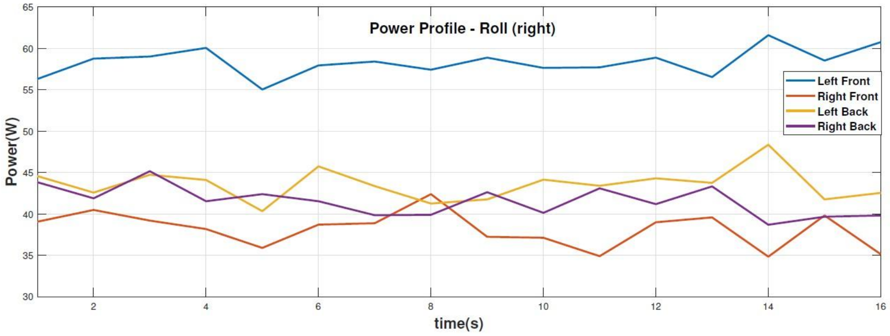


Fig 5 - DJI Air 2 forward flight with a right roll

Field Experiments using DJI Air 2

Power comparison based on maneuvers

Table 2 : Avg Power for various maneuvers using DJI Air 2

Maneuver	Motor Power (W)				Total Power(W)
	Left Front	Right Front	Left Back	Right Back	
Hover	38.01	42.16	33.37	31.8	145.34
Pitch(F)	38.09	42.26	52.32	53.53	186.2
Pitch(B)	49.31	52.76	24.76	28.05	154.88
Roll(L)	34.26	57.04	36.03	43.67	171
Roll(R)	58.12	38.22	43.67	41.72	181.73
Ascend	57.24	50.36	42.2	40.7	190.5
Descend	42.7	35.3	31.7	27.09	136.79
Ascend+Pitch(B)	69.59	70.21	39.33	39	218.13
Descend+Pitch(F)	27.87	30.73	31.06	31.84	121.5
Yaw(CL)	40.14	45.81	34.66	34.58	155.19

Figure 6 - DJI Air 2 drone during hover
Location - EEE Department, IIT Guwahati

Table 3 - DJI Air 2 specifications

Mass	570 gms
Dimension	183x253x77 mm
Max Flight time	34 minutes
Battery Capacity	3500 mAh
Maximum Range	18.5 kms
Max Ascend Velocity	4 m/s
Max Horizontal Velocity	12 m/s (N Mode) 19 m/s (S Mode)

Field Experiments using DJI Air 2

Power comparison with a power model in [1]

Induced Power

$$P_i = k_1 T \left(\sqrt{\frac{T}{2\rho A} + \left(\frac{V_{vert}}{2}\right)^2} + \frac{V_{vert}}{2} \right)$$

Profile Power

$$P_p = \sum_{i=1}^{M} P_{p,i} = \sum_{i=1}^{M} \left(\frac{N \times c \times c_d \times \rho \times R^4}{8} {\omega_i}^3 \left(1 + {\mu_i}^2\right) \right)$$

$$P_p = \sum_{i=1}^{M} \left(\frac{N \times c \times c_d \times \rho \times R^4}{8} \left({\omega_i}^3 + \left(\frac{V_{air} cos(\alpha_i)}{R} \right)^2 \omega_i \right) \right)$$

$$P_{p,hover,i} = \frac{N \times c \times c_d \times \rho \times R^4}{8} {\omega_i}^3$$

$$\mu_i = \frac{V_{air} cos(\alpha_i)}{\omega_i R}$$

Parasitic Power

$$P_{par} = \frac{1}{2} C_d \times \rho \times A_{quad} \times {V_{air}}^3$$

Experiments

$$P_{exp1} = P_{i,hover}(mg, 0) + P_p(mg, 0) = (c_1 + c_2)(mg)^{\frac{3}{2}}$$

$$P_{exp2} = P_i(mg, V_{vert}) + P_p(mg, 0)$$

$$\begin{aligned} P_{exp3} &= P_i(T, 0) + P_p(T, V_{air}) + P_{par}(V_{air}) \\ &= (c_1 + c_2) T^{\frac{3}{2}} + c_3 (V_{air} cos\alpha)^2 T^{\frac{1}{2}} + c_4 V_{air}^3 \\ &\simeq (c_1 + c_2) T^{\frac{3}{2}} + c_4 V_{air}^3 \end{aligned}$$

T : Total thrust applied by the UAV
k_1 : Ratio of actual airflow to idealised uniform airflow
ρ : Density of air
A : Total propeller area
V_{vert} : Vertical velocity of the UAV
V_{air} : Horizontal velocity of the UAV
N : Total number of blades in a single propeller
M : Total number of rotors
c_d : Drag coefficient of the blade
c : Blade chord width
C_d : Drag coefficient of vehicle body
R : Radius of the propeller blade
ω_i : Angular speed of i^{th} rotor
μ_i : Advance ratio for propellers in rotor i
α_i : Angle of attack for propeller disks in rotor i
V_{wind} : Velocity of wind head on to the UAV
V_{ground} : Ground velocity of the UAV
A_{quad} : Cross sectional area of the vehicle when against wind
c_l : Lift coefficient

[1] Liu et. al., “A power consumption model for multirotor small unmanned aircraft systems,” in *2017 ICUAS*. IEEE, pp. 310–315.

Field Experiments using DJI Air 2

Power comparison with a power model in [1]

$$P_{exp1} = P_{i,hover}(mg,0) + P_p(mg,0) = (c_1 + c_2)(mg)^{\frac{3}{2}}$$

$$P_{exp2} = P_i(mg, V_{vert}) + P_p(mg,0)$$

$$P_{exp3} = P_i(T,0) + P_p(T, V_{air}) + P_{par}(V_{air})$$
$$= (c_1 + c_2)T^{\frac{3}{2}} + c_3(V_{air}cos\alpha)^2 T^{\frac{1}{2}} + c_4 V_{air}^3$$
$$\simeq (c_1 + c_2)T^{\frac{3}{2}} + c_4 V_{air}^3$$

Power model coefficients for DJI Air 2

Parameter	Value	Parameter	Value
m	0.57 kg	c2	9.02 $(m/kg)^{1/2}$
g	9.8 m/s2	c3	~0
k1	2.4795	c4	-0.033611 kg/m
k2	1.2346 $(kg/m)^{1/2}$	c5	-0.0048941 Ns/m
c1	1.99 $(m/kg)^{1/2}$	c6	~0

[1] Liu et. al., "A power consumption model for multirotor small unmanned aircraft systems," in *2017 ICUAS*. IEEE, pp. 310–315.

Field Experiments using DJI Air 2

Power comparison with a power model in [1]

Actual Power vs Power model estimate from [1]

Maneuver	(Horizontal velocity, Vertical Velocity) (m/s, m/s)	Total Power (W)	Power from Liu et al. model (W)	Power Difference (W)	% Power difference
Hover	(0,0)	145.34	145.35	-0.01	-0.00
Pitch(F)	(11.9,0)	186.2	186.86	-0.66	-0.35
Pitch(B)	(11.98,0)	154.88	187.86	-32.98	-21.29
Roll(L)	(12.01,0)	171	188.25	-17.25	-10.08
Roll(R)	(11.1,0)	181.73	177.77	3.96	2.18
Ascend	(0,4.14)	190.5	186.18	4.32	2.27
Descend	(0,-3.02)	136.79	131.87	4.92	3.59
Ascend + Pitch(B)	(8.61,3.99)	218.13	203.33	14.8	6.78
Descend + Pitch(F)	(11.1,-4.9)	121.5	152.87	-31.37	-25.81
Yaw(CCL)	(0,0)	156.15	145.35	10.8	6.91
Yaw(CL)	(0,0)	155.19	145.35	9.84	6.34
Pitch(F)	(16.38,0)	297.44	273.52	23.92	8.04
Pitch(B)	(16.82,0)	230.9	285.78	-54.88	-23.76
Roll(R)	(18.4,0)	262.96	336.06	-73.1	-27.79
Roll(L)	(18.94,0)	244.67	355.58	-110.91	-45.33
Ascend + Pitch(B)	(15.14,4.02)	275.64	310.55	-34.91	-12.66
Descend + Pitch(F)	(17.95,-5)	187.37	270.18	-82.81	-44.19

[1] Liu et. al., "A power consumption model for multirotor small unmanned aircraft systems," in *2017 ICUAS*. IEEE, pp. 310–315.

Conclusion

- Power model based on maneuvers helps in estimating instantaneous power in each rotor.
- The model helps us differentiate trajectories based on power.
- Field experiments on DJI Air 2 drone reveals the power differences with various maneuvers.

Future Work

- Power model based on non-linear dynamics of the quadrotor.
- Calculating parameters of a drone to test our power formulation.
- Mapping rotor speed with current for a practical quadrotor motor.
- Formulating a realistic path planning scenario.
- Multi-agent setup.

Publications

- Paraj Ganchaudhuri, Chayan Bhawal, “Power consumption of a quadrotor based on maneuvers,” *to be published in IEEE-GCON 2023,* 23-25 June 2023, Guwahati, India.

Thank you