PMRF REVIEW REPORT

Path Planning of a swarm of UAVs with Minimal Energy Consumption

1 Introduction

Unmanned Aerial Vehicles (UAVs) or drones have revolutionized modern transportation system for delivery and surveillance operations. One of the well-known problems concerning UAVs is the path planning of a UAV with minimal energy consumption. Formulation of such a problem in a mathematical framework requires a model that can predict the power consumed by a UAV during flight. Note that power consumed by an UAV depends on multiple parameters and these parameters are not easy to identify and track in general. As a result, there are inaccuracies in predicting the power consumed by a UAV in-flight. In this research, we aim to address three aspects concerning path planning of a quadrotor UAV. First, we want to derive a Power Consumption Model (PoCM) that can accurately predict the power consumed by a quadrotor in-flight. Second, using the obtained PoCM, we aim to perform path planning given an operation/objective such that the energy consumed in a trip is minimized. Third, we plan to devise an algorithm for path-planning in a multi-agent setup and dynamically changing environment such that a given objective is achieved with minimum power consumed by each agent.

In this phase of work, we have primarily focussed on conducting an elaborate literature review to gauge the gap in literature pertaining to PoCM of a UAV. Based on this review, we could segregate different PoCMs of UAVs into modelling based on: power components, aerodynamic aspects, Helicopter dynamics, and flight experiments. We inferred that there is a gap in the literature on a unanimous PoCM for a quadrotor. Hence, in the next stage of our work, we plan to derive a PoCM based on the angular speed of a quadrotor's motors. We elaborate on the theory and approach behind our plan in the next section.

2 Power Consumption Modelling of a quadrotor UAV

There are a bunch of parameters that affect power consumption of a quadrotor like weight, payload, number of rotors, air density, airspeed, altitude, etc [1]. In spite of having different PoCM for quadrotors, there is no consensus among these PoCMs on the power consumed by a quadrotor for a given operation [2].

To solve the problem at hand, we aim to develop a PoCM that takes into account the physical attributes of a quadrotor, the environment in which it flies and the manoeuvrers it undertakes and maps these to a single parameter, i.e., the angular speeds of the rotors/propellers. The underlying idea behind this approach is the fact that rotors are the end consumers of power in a quadrotor. Using the angular speeds, we aim to obtain a PoCM that can calculate the energy required for a trip.

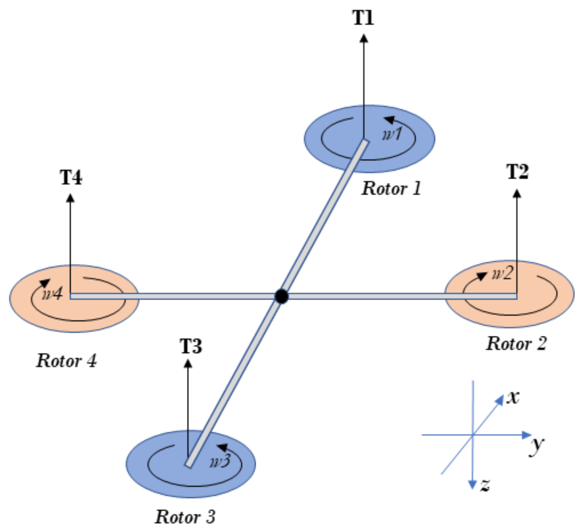

Figure 1: Quadrotor with their thrust vectors and directions of rotation of propellers/rotors.

Next, we elaborate on the parameters that affect thrust and subsequently relate how thrust varies with angular speed of rotors.

2.1 Parameters affecting Thrust

Note that a quadrotor generates thrust using the propellers connected to rotors. The vertical component of this thrust is known as lift which balances the weight of the quadrotor, while the horizontal component counteracts the drag and helps in translational motion: see Figure 1. The parameters that affect thrust are:

- **Fixed:** No. of blades in a single propeller, Drag coefficient of blade, Blade chord length, Angle of attack of propeller blade, Propeller blade area.
- **Variable:**
  - **Drone Design:** Drone weight (fixed with fixed battery), Payload weight.
  - **Environment:** Wind velocity, Wind incident angle, Air density, Gravity.
  - **Drone dynamics:**
    - Climb/Descend (Vertical motion) - All propellers produce the same thrust. Climb/sink rate (vertical velocity) decides the required thrust dynamics.

* Hover - All propellers produce the same thrust.
* Roll - Right side and left side propellers produce unequal thrust producing a rolling torque. Roll angle decides the required thrust in respective propellers.
* Pitch - Front and rear propellers produce unequal thrust producing a pitching torque. Pitch angle decides the required thrust in respective propellers.
* Yaw - Clockwise and counter-clockwise spinning propellers produce unequal torques resulting in a yaw torque. Yaw angle decides the required thrust in respective propellers.
* Cruise (Horizontal motion) - Combination of pitch and hover. The thrusts are unequal due to pitch. The horizontal velocity depends on the pitch angle.

As motivated in the beginning, we aim at formulating a PoCM based on the angular speed of the rotors. To this end, in the next section we elaborate on the relation between thrust and angular speed of rotors.

2.2 UAV dynamics with Thrust and Angular velocity of rotors

Let w be the angular velocity in number of revolutions per minute (RPM) of the rotor and b be a parameter that depends upon the fixed parameters (as mentioned above) and air density. Then, thrust T is given by: $T = bw^2$.

Pairwise differences in rotor thrusts cause the vehicle to rotate. The torque about the vehicle's x-axis, the rolling torque, is generated by the moments $\tau_x = d(T_4 - T_2)$ where d is the distance from the rotor axis to the centre of mass. Using the expression for thrust, we have

$$\tau_x = db(w_4^2 - w_2^2) \tag{1}$$

Similarly, the pitching torque, i.e., the torque about the vehicle's y-axis is $\tau_y = db(w_1^2 - w_3^2)$.

The torque applied to each propeller by the motor is opposed by aerodynamic drag given by $Q_i = cw_i^2$ and $i \in \{1, 2, 3, 4\}$. c depends on the same factors as b. This torque exerts a reaction torque on the airframe which acts to rotate the airframe in the opposite direction to its rotation. The reaction torque about the z-axis is

$$\tau_z = Q_1 - Q_2 + Q_3 - Q_4 = c(w_1^2 - w_2^2 + w_3^2 - w_4^2)$$

So, a yaw torque is generated by appropriate coordinated control of all four rotor speeds.

We can write the thrusts and torques acting on the quadrotor as

$$\begin{bmatrix} T \\ \tau \end{bmatrix} = \begin{bmatrix} -b & -b & -b & -b \\ 0 & -db & 0 & -db \\ db & 0 & -db & 0 \\ c & -c & c & -c \end{bmatrix} \begin{bmatrix} w_1^2 \\ w_2^2 \\ w_3^2 \\ w_4^2 \end{bmatrix} = A \begin{bmatrix} w_1^2 \\ w_2^2 \\ w_3^2 \\ w_4^2 \end{bmatrix} = AW^2 \tag{2}$$

The matrix A is constant and full rank if $b, c, d > 0$ and we can obtain the required rotor speeds using $W = \left(A^{-1} \begin{bmatrix} T \\ \tau \end{bmatrix} \right)^{\frac{1}{2}}$. Using eq(2), we get a map from thrust and torque to required rotor speeds.

In order to map the flight dynamics to the rotor speeds, we need to know the controller dynamics. This is described next.

2.3 Control architecture for translating forward flight into rotor speeds

Note that in a quadrotor, the thrust to be generated by each of the rotors are regulated by the angular speeds of the rotors. The angular speed, on the other hand, is controlled by the controller used to rotate the rotors to obtain the desired thrust. Hence, we assume that a PID controller is used to regulate the thrust produced by the rotors.

For altitude, let z^* and $z^{\#}$ be the desired and actual altitudes respectively, K_p and K_d be the controller gains and $T_0 = mg$ to be the weight of the vehicle. Then thrust required for attitude control is

$$T_z = K_p(z^* - z^{\#}) + K_d(\dot{z}^* - \dot{z}^{\#}) + T_0 \tag{3}$$

We can determine the average rotor speeds from eq(3) using $T = bw^2$.

For pitch and x-translational motion, a PD controller is used to compute the pitching torque. Let θ_p^* and $\theta_p^{\#}$ be the desired and actual pitch angles, respectively and $K_{\tau,p}$, $K_{\tau,d}$ be the controller gains. The pitching torque is given by

$$\tau_y = K_{\tau,p}(\theta_p^* - \theta_p^{\#}) + K_{\tau,d}(\dot{\theta_p}^* - \dot{\theta_p}^{\#}) \tag{4}$$

Here, pitch angle θ_p depends on the required translational velocity.

Similarly, we have PID controllers for roll and yaw actions. So for every maneuver that the quadrotor undertakes, there exists a roll, pitch, yaw torque and average thrust (to balance the weight of the quadrotor). Using the values of torques and thrust, we can estimate the required rotor speeds.(using eq(2))

2.4 Power Calculation

In this section, we show the relation between thrust and power consumed. The shaft power for a DC motor drive is given by

$$P = \frac{2\pi}{60} \times w \times T_{or} \tag{5}$$

where w is the angular speed of the motor in RPM, T_{or} is the torque in $N - m$ and Power P is in Watts. According to Disk actuator theory, the torque produced by a rotor is given by $T_{or} = \frac{1}{2} T_q \rho A w^2 R^3$, where T_q is the torque coefficient that is fixed for a given propeller, ρ is air density, A is propeller disk area and R is blade chord length.
Considering $T_{or} \propto w^2$ and using eq(5), we have $P \propto w^3$.
Here ρ is assumed constant for an operation while the remaining parameters are constant by mechanical construction of the quadrotor. Therefore, we make an attempt to map every dynamics and every parameter that affects thrust to a single variable w for obtaining the power consumption profile of an quadrotor.

2.5 Simulations

To test the above formulation, we perform simulations of an quadrotor motion using MATLAB. We give commands to a quadrotor in terms of Cartesian coordinates and then obtain the angular speeds of all four rotors. Here, we have a trip as a sequence of altitude changes. Initial position = (0,0,0). Target Position = (0,0,1) at time $t = 0$s, (0,0,2) at time $t = 5$s, (0,0,3) at time $t = 10$s. The angular speed profiles are shown in Figure 2. The power profiles will be proportional to the cube of the angular speed profiles. Integrating the power profiles, we can estimate the energy that will be required for this particular trip.

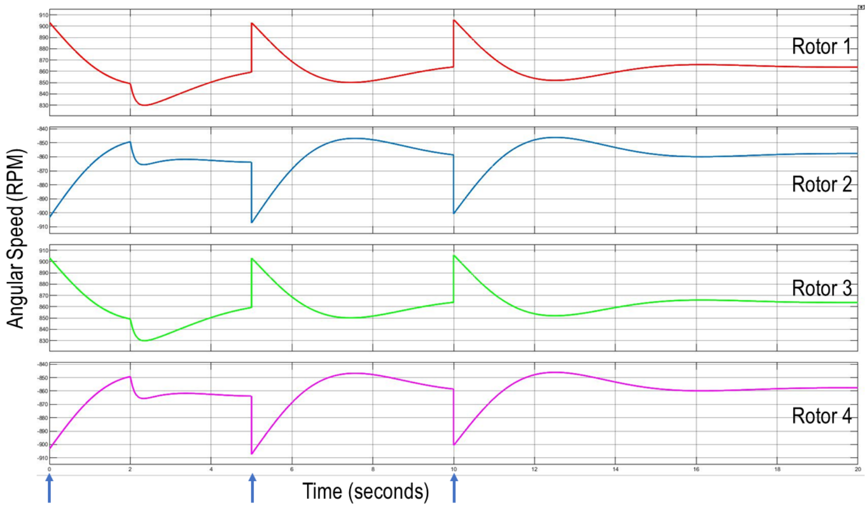

Figure 2: Angular speed profiles of individual rotors of the quadrotor

Note that since the power consumed depends on the angular speed of the rotors and since the angular speed of the rotors is a time-dependent parameter, power consumed by a quadotor is clearly a time-dependent parameter. To the best of our knowledge, all available PoCMs in the literature, predict power as a time-independent parameter, i.e., such models spit out power as a static number. Based on the literature survey and simulations carried out, we infer that considering power consumed by a quadrotor to be time-independent is what leads to an inaccurate prediction of energy consumed by the quadrotor in a trip.

3 Conclusion

A trajectory or path of an quadrotor is a set of maneuvers. For example, a straight-line path requires a pitching maneuver (pitch angle depends on the horizontal velocity). Similarly, any curved line path requires a sequence of roll, pitch and yaw maneuvers. In the next phase, we aim to quantify every instant of the path as a set of roll, pitch and yaw angles and the average thrust required. Using these, we plan on extracting the required rotor speeds from which we can derive a dynamic power profile that will help accurately predict the energy consumed for a trip. A successful implementation of this will help us move one step closer to the path planning aspects of a set of quadrotors.

References

[1] Z. Liu, R. Sengupta, and A. Kurzhanskiy, "A power consumption model for multi-rotor small unmanned aircraft systems," in *2017 International Conference on Unmanned Aircraft Systems (ICUAS)*. IEEE, 2017, pp. 310–315.

[2] J. Zhang, J. F. Campbell, D. C. Sweeney II, and A. C. Hupman, "Energy consumption models for delivery drones: A comparison and assessment," *Transportation Research Part D: Transport and Environment*, vol. 90, p. 102668, 2021.